Simulation on separation flowing around cylinders with lattice Boltzmann method

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Abstract

In computational fluid dynamics, traditional methods show low rate of convergence on low speed flow, while lattice Boltzmann method performed well on it. In this report, we use lattice Boltzmann method to simulate cylinder flow, which cylinders are in many shapes. These results will be important to further study on control of cylinder flow.

Keywords: cylinder flow, separation flow, lattice Boltzmann method

Introduction

Lattice Boltzmann method (LBM) has been widely used in computational fluid dynamics (CFD), different from other traditional methods, which need to solve Jacobian matrix, LBM solves Boltzmann method with single variable particle distribution function $f(\mathbf{x},\xi,t)$ instead of Navier-Stokes equations. When simulating low speed flow, traditional methods show low rate of convergence. LBM is explicit scheme, which can solve equations fast. Normally, LBM uses Cartesian coordinate. When treating with curved boundaries, lattice will be broken into different parts. It is hard to describe the curved boundaries. At very first, A.J.C.Ladd (1994) suggests to use Link Bounce-Back scheme to treat the curved boundaries, but this treatment will change the curved boundaries into "coarse lattice boundary". Later, O.Flilppova and D.Hänel (1998) propose a new boundary treatment for curved boundaries. But the stability is not so good. Then R.Mei et al. (1999) improved its stability, but when treating with low Reynolds flow, the stability is still not enough. In 2002, Z.L.Guo et al. (2002) proposed an extrapolation method for curved boundaries. Lately, Z.D. Wang et al. (2013) proposed a new extrapolation treatment, which improve the accuracy and stability in low Reynolds flow.

Governing Equation

Lattice Boltzmann method solves the following discretization equation:

$$f_i(x_\alpha + c_{i\alpha}\delta t, t+\delta t) = f_i(x_\alpha, t) - \omega\left[f_i^{eq}(x_\alpha, t) - f_i(x_\alpha, t)\right] \tag{1}$$

where f is the density distribution function of particles, $c_{i\alpha}$ is the discretized velocities and α represents Cartesian coordinate. We simulate the low speed flow using D2Q9 model, which is on of the DdQb models proposed by Y.H. Qian et al. (1992). The equilibrium equation is chosen as follows:

$$f_i^{\mathrm{eq}}(\boldsymbol{x},t)=w_i\rho\left\{1+\frac{c_{i\alpha}u_\alpha}{c_s^2}+\frac{u_\alpha u_\beta}{2c_s^2}\left(\frac{c_{i\alpha}c_{i\beta}}{c_s^2}-\delta_{\alpha\beta}\right)\right\} \qquad (2)$$

where ρ and u_α represent the density and velocity of the fluid particle at position $\boldsymbol{x}$ and time t, c_s is the speed of sound, the index i denotes different particles' dimensionless velocity and $\delta_{\alpha\beta}$ is the Kronecker operator. w_i and c_s is chosen as

$$w_i=\begin{cases}\frac{4}{9}, & i=0,\\ \frac{1}{9}, & i=1\sim 4,\\ \frac{1}{36}, & i=5\sim 8\end{cases} \qquad c_s=\frac{1}{\sqrt{3}}\frac{\delta x}{\delta t}. \qquad (3)$$

To simulate the Newtonian fluid, ω is related to the shear viscosity, which can be driven by Chapman-Enskog expansion as

$$\frac{1}{\omega}=\frac{\nu}{c_s^2\delta t}+\frac{1}{2}. \qquad (4)$$

Boundary Condition

On the boundary, we use the boundary treatment proposed by WANG et al. This boundary treatment divide the fictitious particle distribution into the equilibrium part and nonequilibrium part as follows:

$$f_i(\boldsymbol{x}_b,t)=f_i^{(\mathrm{eq})}(\boldsymbol{x}_b,t)+f_i^{(\mathrm{neq})}(\boldsymbol{x}_b,t) \qquad (5)$$

where $f_i^{(\mathrm{eq})}(\boldsymbol{x}_b,t)$ is set as

$$f_i^{(\mathrm{eq})}(\boldsymbol{x}_b,t)=w_i\rho(\boldsymbol{x}_b,t)\left[1+\frac{c_{i\alpha}u_{b,\alpha}}{c_s^2}+\frac{(c_{i\alpha}u_{b,\alpha})^2}{2c_s^4}-\frac{u_b^2}{2c_s^2}\right] \qquad (6)$$

where $\rho(\boldsymbol{x}_b,t)=2\rho(\boldsymbol{x}_f,t)-\rho(\boldsymbol{x}_{ff},t)$ and $\boldsymbol{u}_b=\frac{2\lambda}{1+\lambda}\boldsymbol{u}_f-\frac{\lambda}{2+\lambda}\boldsymbol{u}_{ff}$.

For the nonequilibrium part, we define $f_i^{(\mathrm{neq})}(\boldsymbol{x}_b,t)$ as follows:

$$f_i^{(\mathrm{neq})}(\boldsymbol{x}_b,t)=2f_i^{(\mathrm{neq})}(\boldsymbol{x}_f,t)-f_i^{(\mathrm{neq})}(\boldsymbol{x}_{ff},t). \qquad (7)$$

Simulations

In this report, polygonal cylinders and oval cylinders with different eccentricities are simulated. Because of the resolution, the critical flow when it separated cannot be simulated. To find the critical Reynolds number, higher Reynolds number flows are simulated and extrapolated to critical Reynolds number.
The Reynolds number is define as:

$$\mathrm{Re}=\frac{UL}{\nu} \qquad (8)$$

In flows around polygonal cylinders, L is the diameter of their circumcircles. In flows around oval cylinders, L is the length of major axis.

As experiment assembled by Milton Van Dyke (1988) reported, the length of standing eddies increases linearly with Reynolds number. The results of simulations are shown below:

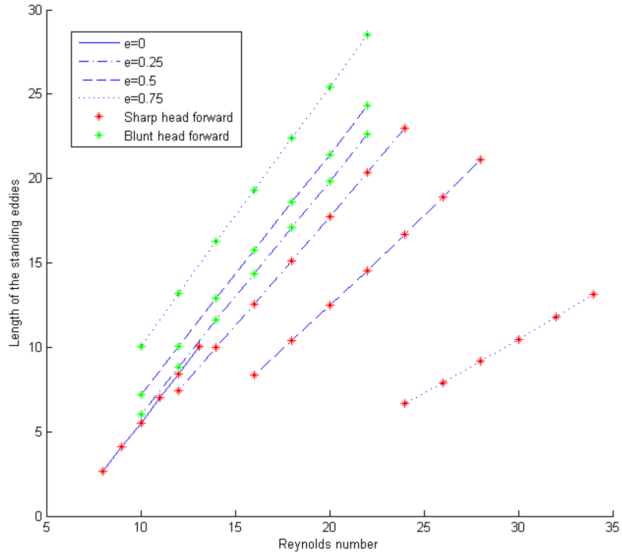


Figure 1. Eddies' lengths of ovals

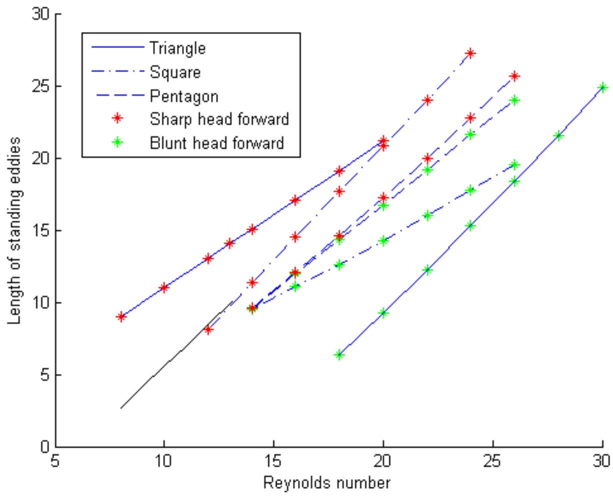
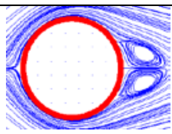
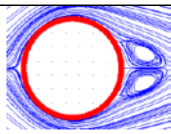
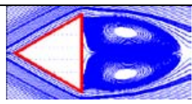
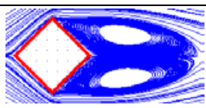
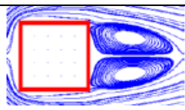
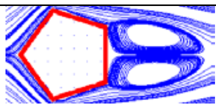
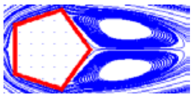
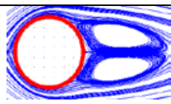
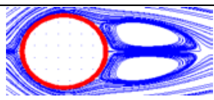
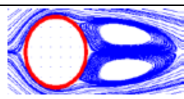
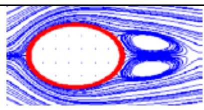
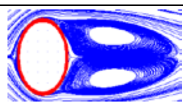


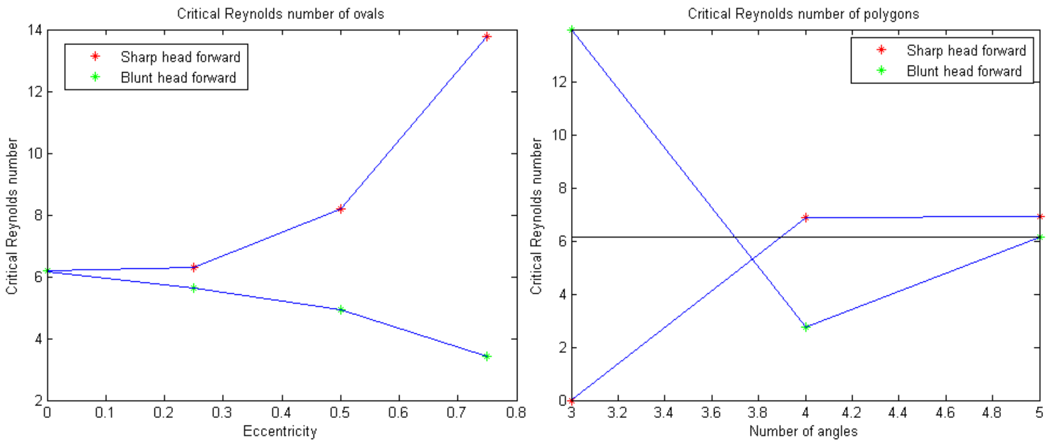
Figure 2. Eddies' lengths of polygons. The black solid line represents circle cylinder

The two figures above show the exact linear relationship between the length and Reynolds number, which agrees with the experiment results. The slopes of these straight lines in the figures above are different. It shows how separating vortex changes after separated. We consider it is relative to aspect ratio of each cylinder. The slope increases as the aspect ratio increases.
The critical Reynolds number is shown in the table below:

Table 1. Critical Reynolds number and streaming pattern

	Sharp head forward	Blunt head forward	Streamlines pattern (sharp head forward)	Streamlines pattern (blunt head forward)
Circle	6.1758	6.1758		
Triangle	0	13.9904		
Square	6.9117	2.7690		
Pentagon	6.9461	6.1410		
Oval (e=0.25)	6.2908	5.6197		
Oval (e=0.5)	8.2046	4.9342		
Oval (e=0.75)	13.7695	3.4372		

The variation tendency of oval cylinders is shown below:



(a) (b)

Figure 3. Tendency of critical Reynolds numbers

When sharp head point forward, the tendency of critical Reynolds number of oval cylinders monotone increases as the eccentricity increases. When blunt head point forward, critical Reynolds number monotone decrease as the eccentricity decreases. As the number of angles increasing, the critical Reynolds number of polygonal cylinders is approaching that of circle cylinder (the black line in Figure 3(b)).

Conclusions

We successfully simulated the flows around various cylinders and get some preliminary result. We find the separation flow has two important characteristics: critical Reynolds number and the slopes.

Achnowledgment

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