APCOM & ISCM
11-14th December, 2013, Singapore

A Multi-Dimensional Limiter for Hybrid Grid

*** H. W. Zheng [1]**

[1] State Key Laboratory of High Temperature Gas Dynamics, Institute of Mechanics, Chinese Academy of Sciences, Beijing 100190, China

*Corresponding author: h.zheng@imech.ac.cn

Abstract

For the finite volume method, the reconstruction step is employed to obtain the states for the calculation of numerical fluxes at the faces. To remove the non-physical oscillations, a limiting procedure is required. This limiting procedure is so important that it not only influence the numerical accuracy in the smooth regions but also affect the robustness of the solver. For the unstructured meshes, the design of the limiting procedure is not trivial. The well-known and wide-used limiting procedure is proposed by Barth and Jespersen in 1989 and lately improved by Venkatakrishnan in 1993. However, this method is too dissipative and the overshoot or undershoot phenomenon can still be observed. In this paper, a new limiter for hybrid grid is proposed. It limits the state variables of a face directly from the corresponding variables of this face's neighbor cells. It is so simple that it can be easily adopted in many solvers based on unstructured grid.

Keywords: finite volume, multi-dimensional limiter, hybrid grid.

Introduction

A main computational challenge with nonlinear hyperbolic equations is the resolution of discontinuities. However, any linear scheme higher than first order accuracy cannot generate monotonic solutions. Hence, the non-linear limiting function is introduced to avoid numerical oscillations [1-5].

A good limiting function should be able to remove the non-physical oscillations nearby the shock and can also preserve the numerical accuracy in the smooth regions. Moreover, the limiter function should not affect the convergence to the steady state. Barth and Jespersen (1989) introduced the first limiter for unstructured grids. The Barth and Jespersen limiter is used to enforce a monotone solution. The main idea of their work is to avoid introducing oscillation is that no new local extrema are formed during reconstruction. The scheme consists of finding a value in each control-volume that will limit the gradient in the piecewise linear reconstruction of the solution. However, their method is rather dissipative which leads to smear discontinuities. Furthermore, the limiter may be active in smooth flow regions due to the numerical noise, which causes difficulties for steady state convergence. To improve the convergence, Venkatakrishnan (1993) proposed a smooth differentiable alternative of the minimum function in Barth-Jespersen. However, it does not preserve strict monotonicity, slight oscillations can be observed near shock discontinuities. Moreover, similar to Barth and Jespersen (1989), it is quite dissipative that predicted accuracy also cannot be guaranteed with the fixed stencil when these limiters are used.

In this work, a new multidimensional limiting procedure is proposed to limit the gradient in each direction independently. For each face of one cell, only the gradient along the direction between the two centroids is limited. This will reproduce a limited difference for each face. After having these limited differences, the unlimited differences and the limited differences for each face are limited secondly. This produces a new limited difference which is multidimensional in its very nature. The rest of paper is organized as follows. The governing equation and numerical method are described in section 2 and section 3 respectively. The numerical result and discussion is presented in section 4. The final section gives a summary about the main work of the paper.

Governing equations

In this paper, the steady Farve-averaged Navier–Stokes equations with the two equation k-ω turbulence model are considered,

$$\partial_t \int_{\Omega} U dV + \int_{S} F\left(\mathrm{U}, \vec{\mathrm{n}}\right) dS = 0 \tag{1}$$

$$\partial_t \int_{\Omega} \Xi dV + \int_{S} \mathrm{H}\left(\Xi, \vec{\mathrm{n}}\right) dS = \int_{\Omega} \Theta dV \tag{2}$$

Where U and Ξ are the state vectors, F and H are the normal flux vectors. They can be expressed as

$$U = \begin{pmatrix} \rho \\ \rho \mathrm{u} \\ E \end{pmatrix}, F = F^c - F^\upsilon, F^c = u_n U + P \begin{pmatrix} 0 \\ \mathrm{n} \\ u_n \end{pmatrix}, F^\upsilon = \begin{pmatrix} 0 \\ \left[\tau\right]\cdot \mathrm{n} \\ \left(\left[\tau\right]\cdot u + q\right)\cdot \mathrm{n} \end{pmatrix}, \tag{3}$$

and

$$\Xi = \begin{pmatrix} \rho k \\ \rho \omega \end{pmatrix}, H = H^c - H^\upsilon, H^c = u_n \Xi, H^\upsilon = \begin{pmatrix} \left(\mu_L + \sigma_k \mu_{tur}\right)\nabla k \\ \left(\mu_L + \sigma_\omega \mu_{tur}\right)\nabla \omega \end{pmatrix}\cdot \mathrm{n}, \Theta = \begin{pmatrix} P_k - \rho\varepsilon \\ \alpha\omega P_k / k - \beta\rho\omega^2 \end{pmatrix}. \tag{4}$$

Here, ρ is the density, u is the velocity, E is the total energy, P is the pressure, [I] is the identity tensor, q is the heat flux, [τ] is the stress tensor,

$$\left[\tau\right] = \left(\mu_L + \mu_{tur}\right)\left\{\nabla\vec{u} + \nabla^T\vec{u} - \frac{2}{3}\left(\nabla\cdot\vec{u}\right)\left[\mathrm{I}\right]\right\} \tag{5}$$

Numerical Method and Limiters

The numerical semi-discretization of Eq. (1) is,

$$\frac{\partial}{\partial t}\int_{\Omega} U dV = R \approx \sum_f \left[F_f^c\left(U_f^-, U_f^+, \vec{n}_f\right) - F_f^v\left(U_f^-, U_f^+, \vec{n}_f\right)\right]\cdot A_f \tag{6}$$

To increase the solution accuracy in space, the left and right states are constructed from extrapolated values from cell centers to cell interfaces and then used to construct fluxes (van Lear, 1979),

$$W_f^k = W^k + \Delta^k, \Delta^k = \vec{\nabla} W^k \cdot d\vec{r}^k, \qquad \text{k=–,+} \tag{7}$$

In order to make the solution be monotonic, the slope limiters are enforced in the extrapolation. For example, Barth-Jespersen's reconstruction (Barth and Jespersen, 1989) and limiter reads,

$$W_f^k = W^k + \Delta^{k,\lim}, \Delta^{k,\lim} = \phi^k \vec{\nabla} W^k \cdot d\vec{r}^k, \qquad \text{k=–,+} \tag{8}$$

with,

$$\phi^k = \min_{i \in N(k)} \phi^{ki}, \phi^{ki} = \begin{cases} \psi\left(1.0, \Delta_+ / \Delta_-\right), \Delta_+ = \max_{i \in N(k)} W^i - W^k & \text{if } \Delta_- > 0 \\ \psi\left(1.0, \Delta_+ / \Delta_-\right), \Delta_+ = \min_{i \in N(k)} W^i - W^k & \text{if } \Delta_- < 0 \\ 1.0 & \text{if } \Delta_- = 0 \end{cases} \tag{9}$$

In BJ's method (Barth and Jespersen, 1989), they use a non-differential limiter $\psi(1,y)=\min(1,y)$. This adversely affects the convergence properties of the solver. For this reason, Venkatakrishnan (1993) introduces a smooth alternative of the minimum function in Barth-Jespersen procedure,

$$\psi\left(1.0, \Delta_+ / \Delta_-\right) = \frac{\left(\Delta_+^2 + \varepsilon^2\right) + 2\Delta_- \Delta_+}{\Delta_+^2 + 2\Delta_-^2 + \Delta_- \Delta_+ + \varepsilon^2} \tag{10}$$

Although the Venkatakrishnan limiter is used to prevent the non-physical oscillations nearby the shock region, the overshoot or undershoot phenomenon can still be observed. Moreover, the numerical accuracy is degraded by using Venkatakrishnan limiter. Besides, it could be easily observed that the gradient in Barth-Jespersen's and Venkatakrishnan version is limited by multi-plying a scalar limiter ϕ. That is, the limiter is the same for each direction. This shows they may be too dissipative.

Hence, in this paper, a new multidimensional limiting procedure is proposed to limit the gradient in each direction independently. The main idea is to limit the gradient normal to the face direction for each face. For each face of one cell, only the difference along the direction between the two centroids is limited,

$$\Delta^{k,\lim} = \chi^k L\left(\Delta^{k,-}, \Delta^{k,+}\right) \tag{11}$$

with

$$\Delta^{k,-} = 2\vec{\nabla} W^k \cdot \left(\vec{r}^{\,i} - \vec{r}^{\,k}\right) - \left(W^i - W^k\right), \tag{12a}$$

$$\Delta^{k,+} = W^i - W^k . \tag{12b}$$

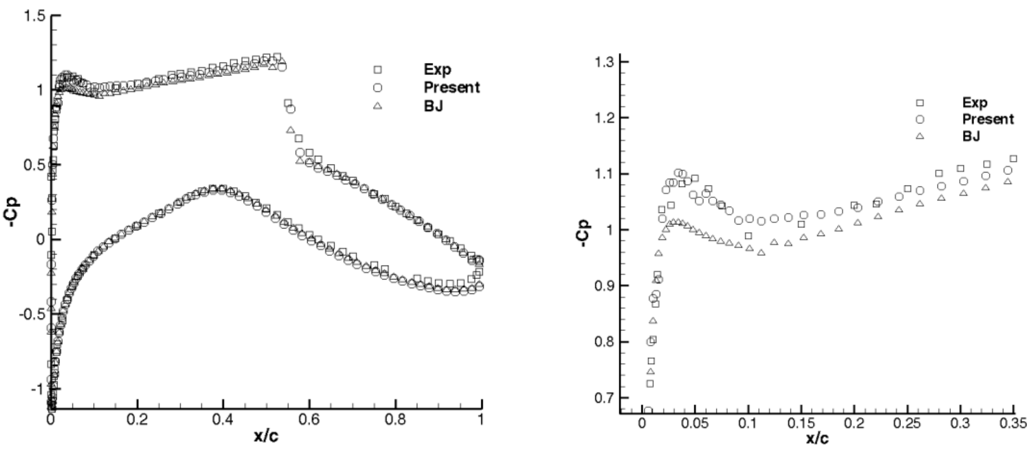
This will reproduce a limited difference for each face. After having these limited differences, the unlimited differences and the limited differences for each face are limited secondly

$$\phi^k = \Upsilon\left(\Delta^{k,\lim}, \Delta^k\right). \tag{13}$$

Results and Discussion

To assess the performance of the proposed limiter, the case of flow around the transonic RAE2822 airfoil is chosen in this section. It is well known that the RAE 2822 airfoil is a supercritical airfoil. The measurements have been done for a variety of flow conditions by Cook et al. (1979). Hence, there are a lot of results for validation.

Here, case number 6 is considered. For this case, the Reynolds number based on a unit chord length is 6.5 million, the Mach number is 0.729, and the angle of attack is 2.31 degrees. It is a transonic speed case with a thin boundary layer attached to the aerofoil surface. The flow separation is not expected. In this particular case, the results are expected to be primarily dictated by the RANS solver. The first grid spacing off the wall is about $1 \cdot 10^{-5}$ chord length. The flow is assumed to be fully turbulent, i.e. transition is not specifically imposed. For the presented simulation, the Green-Gauss reconstruction technique is employed. The convective fluxes are based on the ROE scheme. A viscous wall boundary condition used over the surface of the airfoil, and a free-stream boundary condition used at the outer edge of the domain where the flow is everywhere subsonic. The flow is initialized using free stream values.



(a) (b)

Figure 1. A plate

The distributions of the pressure coefficient and of the friction coefficient on the airfoil surface are plotted in figures 1 and 2. Both results agree well with the experimental data of Cook et al. (1979).

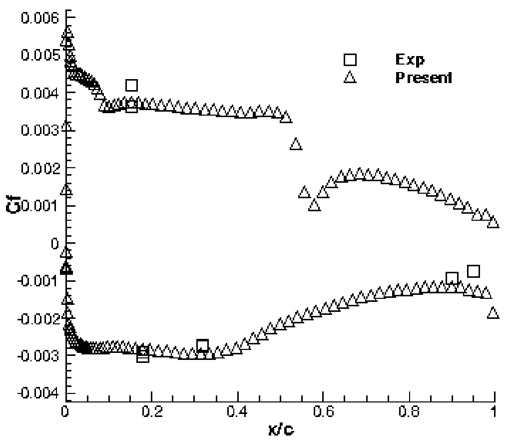


Figure 2. A plate

Besides, from Fig. 1, it is found that the distribution of Cp by present limiter is better than that by BJ limiter as compared to the experimental result. This can be clearly observed in Fig. 1 (b). The aerodynamic lift coefficient and the drag coefficient are predicted very accurately. This can be easily found in Table 1. The present results are better than those by BJ limiter as compared with the experimental data. All in all, the results show good performance the proposed limiter.

Table 1. The aerodynamic Coefficients

	CL	CD
Exp	0.743	0.0127
Present	0.753	0.0131
BJ	0.725	0.0144

Conclusions

In this paper, a new limiter for hybrid grid is proposed. It is so simple that it can be easily adopted in many solvers based on unstructured grid. The numerical result around RAE2822 shows that it is less dissipative than BJ method.

Acknowledgement

This research work was supported by the Natural Science Foundation for the Youth of China under Grant 11202220.

References

van Leer, B. (1979), Towards the ultimate conservative difference scheme. V, A second-order sequel to Godunov's Method, J. Comput. Phys., 32, pp101-136.

Venkatakrishnan, V. (1993), On The Accuracy of Limiters and Convergence to Steady State Solutions, Technical Report AIAA-93-0880.

Barth, T. J. and Jespersen,D.C. (1989), The design and application of upwind schemes on unstructured meshes,AIAA Paper, 89-0366.

Cook, P.H., McDonald, M.A., and Firmin, M.C.P. (1979), Aerofoil RAE 2822 - Pressure Distributions, and Boundary Layer and Wake Measurements, Experimental Data Base for Computer Program Assessment, AGARD Report AR 138.