Random Vibration Analysis for Impellers of Centrifugal Compressors Through the Pseudo-Excitation Method

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Abstract

Impellers of large centrifugal compressors are loaded by fluctuating aerodynamic pressure in operations. Excessive vibration of the impeller can be induced by unsteady airflows which may cause fatigue failures. Traditional vibration analyses require multiple load-step computations with input of pneumatic force in the time domain which are usually very time consuming. Hence, it is necessary to develop random vibration models and solve them in the frequency domain. In this paper, the finite element model is generated based on the result of unsteady CFD analysis for an unshrouded impeller. The pseudo-excitation method is used to obtain the power spectra density of the three-dimensional, dynamic displacement and stress of the impeller. Compared with the direct transient vibration analyses in time domain, the pseudo-excitation method provides accurate and fast estimation of dynamic response of the impeller, making it an applicable and efficient method for random vibration computation of impellers.

Keywords: Impeller, Random vibration, Aerodynamic load, Pseudo-excitation method

Introduction

Impellers of centrifugal compressors play an important role in generating pressured air and gases for production lines in petroleum and chemical engineering. The impellers are designed to undertake loadings of mass imbalance, centrifugal forces as well as aerodynamic loads. The aerodynamic force provides distributed air pressure on the surface of blades, shroud and hub of the impeller, and is time variant since the condition of the flow of air inside the impeller is generally unsteady or even turbulent. A scheme of transient analysis based on the fluid-solid interaction is required to assess the stress of the impeller to ensure its structural integrity. Two kinds of multiple load-step analyses for the stress of solid and the pressure of fluid are incorporated in the scheme separately, and are combined with numerous rounds of real-time data exchange on the interacting boundary of the solid and the fluid domains (e.g. Bludszuweit 1995, Beckert and Wendland 2001, de Boer et al. 2007, Khelladi et al 2010). The full execution of this scheme is usually very time consuming due to the transient analyses for the impeller. Other options of dynamical stress analysis can be adopted from the standpoint of the frequency domain. Unsteady or turbulent forces can be treated as random loadings with statistic characteristics observed based on temporal records of the force (Tootkaboni and Graham-Brady 2010). In this way, the stress response arisen from the dynamic pressure can be formulated as a problem of random vibration and solved in the frequency domain.

In this paper, the transient stress analysis is performed as a random vibration problem through the Pseudo-Excitation Method (PEM, c.f. Lin and Zhang 2004). Based on an unsteady computational fluid dynamical analysis, the auto power spectral density of the air pressure is obtained. A solid, three dimensional finite element model is created using ANSYS. The random vibration analysis for the impeller is carried out through the PEM using a series of harmonic vibration solutions. A user-defined module incorporated with ANSYS is developed to apply the random loading of multiple dimension and multiple points on the impeller structure. Based on the random vibration analysis, the spectral information of the stress response is obtained. Finally, the structural integrity of the impeller in terms of reliability index is computed. It is shown that the solution efficiency of the PEM-based random vibration is much higher than the conventional multiple step transient stress analysis, and thus is suitable for practical engineering.

Mathematical formulation

The governing equation of motion of the impeller structure excited by unsteady, time variant force based on the finite element method can be written as:

$$\boldsymbol{M}\ddot{\boldsymbol{y}}(t)+\boldsymbol{C}\dot{\boldsymbol{y}}(t)+\boldsymbol{K}\boldsymbol{y}(t)=\boldsymbol{P}(t) \tag{1}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are matrices of mass, damping and stiffness of the impeller. $\boldsymbol{P}$ is the vector of random nodal force applied at each degree of freedom of the model. t is the temporal variable. Herein, it is assumed that the time-invariant loads that contribute only to steady stress have been excluded from the load vector, and P is stationary whose probabilistic function does not change with time.

Based on the PEM, the power spectral density $\boldsymbol{S}_{yy}(\omega)$ can be expressed as

$$\boldsymbol{S}_{yy}(\omega)=\left|\boldsymbol{H}(\omega)\right|^2\boldsymbol{S}_{xx}(\omega) \tag{2}$$

where $\boldsymbol{S}_{xx}(\omega)$ is the power spectral density of a harmonic excitation denoted by $\boldsymbol{x}(t)$ whose frequency is ω; $\boldsymbol{H}(\omega)$ is the matrix of frequency-response function. Substituting a pseudo harmonic excitation vector $\tilde{\boldsymbol{f}}(t)=\sqrt{\boldsymbol{S}_{ff}(\omega)}e^{i\omega t}$ into Eq. (1) leads to a pseudo displacement vector denoted by $\tilde{y}(t)$. It was proved that the auto power spectral density of the displacement $\boldsymbol{y}(t)$ is related to $\tilde{y}(t)$ such that

$$\boldsymbol{S}_{yy}=\left|\boldsymbol{H}\right|^2\boldsymbol{S}_{ff}=\left|\tilde{y}\right|^2 \tag{3}$$

Following Eq. (3), the power spectral densities of strain and stress of each single element can be obtained as well using the conventional finite element method.

In the case of multiple dimensional, multiple pointed, totally coherent excitations, the matrix of auto power spectral density can be written as

$$\boldsymbol{S}(i\omega)=\begin{bmatrix} \boldsymbol{S}_{11}(i\omega) & \boldsymbol{S}_{12}(i\omega) & \cdots & \boldsymbol{S}_{1m}(i\omega) \\ \boldsymbol{S}_{21}(i\omega) & \boldsymbol{S}_{22}(i\omega) & \cdots & \boldsymbol{S}_{2m}(i\omega) \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{S}_{m1}(i\omega) & \boldsymbol{S}_{m2}(i\omega) & \cdots & \boldsymbol{S}_{mm}(i\omega) \end{bmatrix} \tag{4}$$

where m is the number of excitations. The non-diagonal terms in the above matrix are sub-matrices of cross power spectral density that can be expanded along the three coordinate directions. Using the PEM again, the auto power spectral density matrix of displacement response can be obtained as

$$\boldsymbol{S}_{yy}(\omega)=\sum_{j=1}^{m}\tilde{\boldsymbol{y}}_j^*\tilde{\boldsymbol{y}}_j^{\mathrm{T}} \tag{5}$$

where $\tilde{\boldsymbol{y}}_j$ is the j-th pseudo displacement vector generated by pseudo load vector $\tilde{\boldsymbol{x}}_j=\boldsymbol{\varphi}_j^*\sqrt{\lambda_j}e^{i\omega t}$. The subscripts * and T represent complex conjugate and transposition of matrix, respectively. λ_j and $\boldsymbol{\varphi}_j^*$ are the j-th eigenvalue and eigenvector of the auto power spectral density of x.

Assuming that the mean value of the loading is zero, the variance of the displacement response can be derived as

$$\sigma_y^2 = \int_{-\infty}^{+\infty} \boldsymbol{S}_{yy}(\boldsymbol{\omega}) \,\mathrm{d}\,\omega \tag{6}$$

Further, let the probabilistic densities of the structural resistance and loading be both normal (Gaussian) distributed, then the reliability index of structural integrity can be expressed as

$$\beta = (\mu_R - \mu_S) \Big/ \sqrt{\sigma_R^2 + \sigma_S^2} \tag{7}$$

where μ_R and μ_S are mean values of the resistance and loading, and σ_R and σ_S are their standard deviations, respectively. Notice all non-zero, time-invariant stresses resulted from mass imbalance and centrifugal force should be considered when μ_S is computed.

Scheme of PEM-based simulation incorporated with ANSYS

The random vibration analysis is carried out using finite element software ANSYS. A specialized user-defined module is developed to repeatedly run harmonic analysis for solution of auto power spectral densities with every single excitation frequency. The scheme of the numerical simulation can be summarized as follows.

1. Create the finite element model of the impeller and assign basic solution conditions.
2. Compute the matrix of power spectral density based on the given records of the air pressure in the time domain.
3. Specify appropriate range of frequency [$\omega_{\min}$, $\omega_{\max}$] and step size $\Delta\omega$.
4. Let frequency $\omega = \omega_{\min} + i\Delta\omega$. Generate the pseudo excitation and compute the pseudo displacement vector $\tilde{y}_j$ using the harmonic analysis module provided by ANSYS, i.e. (ANTYPE，HARMIC).
5. Superpose for each pseudo displacement to obtain the power spectral density matrix related to frequency ω according to Eq. (5). Compute the power spectral density matrix of the stress field.
6. Repeat steps 4 and 5 until ω is equal to or greater than $\omega_{\max}$.
7. Compute the variances of displacement based on Eq. (6) and of stress at each element through conventional finite element method.
8. Compute the reliability index β according to Eq. (7).

Example

Figure 1 shows the solid model of an unshrouded impeller of a centrifugal compressor. There are nineteen blades evenly distributed in the circumferential direction on the impeller. The whole impeller is meshed with the type solid186 by ANSYS as shown in Figure 2. Considering the axial symmetry of the structure, only a sector of one nineteenth of the model needs to be analyzed. The numbers of node and element for the sector are 7823 and 6283, respectively.

Figure 1. Solid model of the impeller

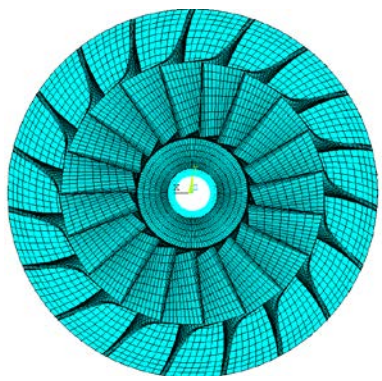

Figure 2. Meshed finite elements

Further, a highlighted area colored in red is illustrated in Figure 3 to show the high stress zone of the impeller based on previous stress analysis using deterministic centrifugal loading. This area is believed the most vulnerable part of the whole impeller to structural failure of fatigue.

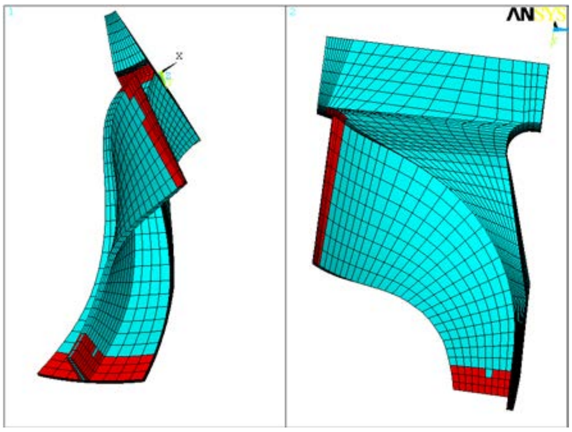

Figure 3. High stress area

The pulsating, unsteady air pressure on the surface of the impeller is obtained through computational fluid dynamics software NUMECA. The time history of the pressure contains 6,000 records and is converted into a three dimensional nodal force of the solid model using the method of radial basis function (Wu, Wang and Sun 2012). Then, the previously presented scheme of PEM-based simulation is carried out to obtain the auto power spectral densities of each nodal point of the impeller. The power spectral characteristics of von Mises stress can be computed as well.
Based on the steady-state stress analysis, the highly stressed area is composed of 200 nodes. Figures 4 and 5 show the auto power spectral densities (PSDs) of x-displacement and von Mises stress of these 200 nodes.

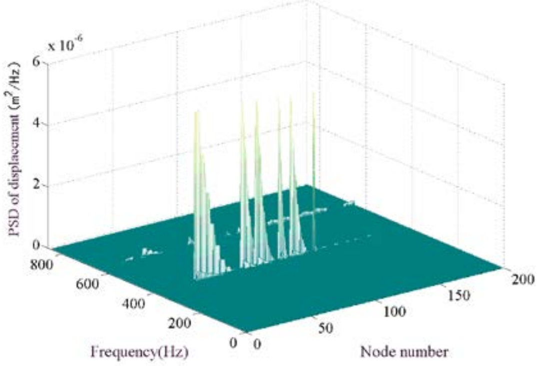

Figure 4. auto PSDs of x-displacement

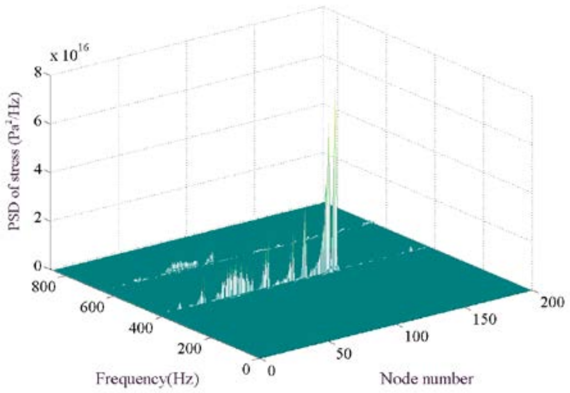

Figure 5. auto PSDs of von Mises stress

Next, the reliability index of the impeller structure is computed following the above analysis for random vibration. Let the rotational speed of the impeller be Ω=5373 r/min, the mean value of stress μ_s can be obtained for each node. The deviance of the nodal stress can be solved using Eq. (6). Assume μ_R=1050MPa, σ_R=0.07, the reliability index of structural integrity can be determined through Eq. (7). The reliability indices of the 200 observed area is presented in Figure 6. The lowest value of the index is 4.79, which means the integrity of these highly stressed positions is guaranteed. By contrast, the indices drop drastically when the rotational speed is increased to 8594 r/min. The lowest value of the reliability index becomes 0.507 in this case, which suggests there will be a structural failure initiating from this position.

It is worth reporting that the solution efficiency of the presented random analysis is satisfactory. In this analysis 6000 records in the time domain and 4096 points of frequency sampling are used. The total computational lasts approximately 8 hours on a four-core workstation. Comparatively, the transient stress analysis in the time domain with 6000 load steps costs about 120 hours. This clearly shows the advantage of dynamical stress computation through the pseudo-excitation method.

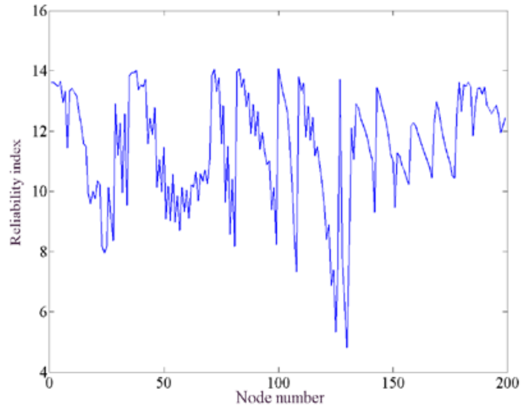

Figure 6. Reliability indices. Ω=5373r/min.

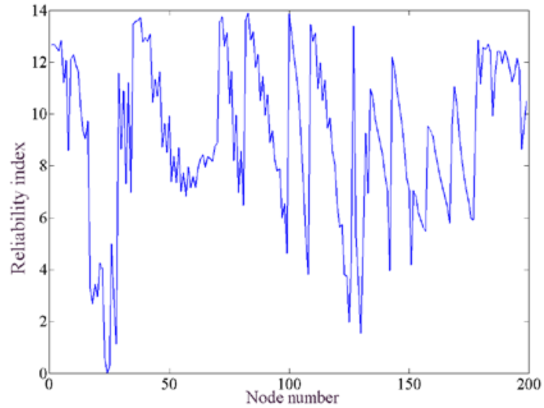

Figure 7. Reliability indices Ω=8594r/min.

Conclusions

It has been demonstrated in this paper that dynamical stress analysis of impellers excited by unsteady aerodynamic force can be dealt with efficiently through random vibration analysis. A PEM based scheme incorporated with ANSYS is developed and executed to obtain the power spectral densities and deviances of stress and displacement. The reliability index of integrity is computed in the case of Gaussian distributions of structural resistance and air pressure. The advantage of the presented scheme in computational efficiency is demonstrated compared with transient stress analysis in the time domain.

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