A Momentum Exchange-based Immersed Boundary-Lattice Boltzmann Method for Fluid Structure Interaction

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Abstract

A new immersed boundary-lattice Boltzmann method (IB-LBM) is proposed and validated in this work by its application to simulate incompressible viscous flows. The conventional IB-LBM based on the lattice Boltzmann equation with external forcing term, which contains a boundary velocity to represent the effect that the moving boundary exerts on the bounce-back distribution functions, whereas the present method use the boundary velocity to compute the momentum exchange in terms of the momentum theorem. Moreover, the momentum exchange based IB-LBM meets the Galilean invariance. Numerical examples show that the present method can provide very accurate numerical results.

Keywords: momentum exchange, immersed boundary, lattice Boltzmann method, fluid structure interaction.

Introduction

In computational fluid dynamics(CFD), a primary issue is the development of accurate, efficient treatments of complex and moving boundaries. Many researchers have developed various numerical methods to resolve this issue. Conventional approaches such as finite difference, finite volume and finite element methods are generally used to accommodate complex geometries with tedious grid generation. However, the recently developed immersed boundary method (IBM) and lattice Boltzmann method (LBM) can handle complex geometry with the use of Cartesian mesh.

The IBM was introduced by Peskin (1977), it can be defined as a non-body-conformal grid method which adds a force density term either explicitly or implicitly to the flow governing equation to satisfy the no-slip condition on the boundary. The adoption of the structured non-body-conformal grid relieves the burden of meshing and reduces the amount of memory and CPU time used compared with unstructured body-conformal grids, and the accurate evaluation of the force density term maintains a high accuracy.

The LBM in previous work by Qian et al (1992) has achieved a great success in simulating complex fluid flows in the past decades. LBM is a particle-based numerical technique, it has two processes: streaming and collision. The major advantage of LBM is its simplicity, easy for implementation, algebraic operation and intrinsic parallel nature. No differential equation and resultant algebraic equation system are involved in the LBM.

In this paper, we couple the immersed boundary method and lattice Boltzmann method, presenting a Galilean invariant momentum exchange equation by introducing the relative velocity into the interfacial momentum transfer to compute the boundary force. The present method is validated by its application to simulate the steady flows past a circular cylinder. The obtained results are compared well with those available in the literature.

Momentum Exchange-based Immersed Boundary-Lattice Boltzmann Method

Lattice Boltzmann model

The Lattice Boltzmann model with single-relaxation time without a forcing term can be written as

$$f_\alpha(\boldsymbol{x}+\boldsymbol{e}_\alpha\Delta t, t+\Delta t) = f_\alpha(\boldsymbol{x},t) - \frac{1}{\tau}[f_\alpha(\boldsymbol{x},t) - f_\alpha^{eq}(\boldsymbol{x},t)] \tag{1}$$

Where $f_\alpha(\boldsymbol{x},t)$ is the density distribution function at position $\boldsymbol{x}$ and time t, f_α^{eq} is its corresponding equilibrium state, τ is the single relaxation parameter, $\boldsymbol{e}_\alpha$ is the particle velocity. For the D2Q9 model as shown in the work by Qian et al (1992), the velocity set is given by

$$\boldsymbol{e}_\alpha = \begin{cases} (0,0) & \alpha=0 \\ c(\cos[(\alpha-1)\frac{\pi}{2}], \sin[(\alpha-1)\frac{\pi}{2}]) & \alpha=1,2,3,4 \\ \sqrt{2}c(\cos[(2\alpha-1)\frac{\pi}{4}], \sin[(2\alpha-1)\frac{\pi}{4}]) & \alpha=5,6,7,8 \end{cases} \tag{2}$$

Where $c = \delta x / \delta t$, δx *and* δt are the lattice spacing and time step. For the case of $\delta x = \delta t$, c is taken as 1. The corresponding equilibrium distribution is

$$f_\alpha^{eq} = \rho w_\alpha [1 + \frac{\boldsymbol{e}_\alpha \bullet \boldsymbol{u}}{c_s^2} + \frac{(\boldsymbol{e}_\alpha \bullet \boldsymbol{u})^2}{2c_s^4} - \frac{u^2}{2c_s^2}] \tag{3}$$

With $w_0 = 4/9$, $w_1 = w_2 = w_3 = w_4 = 1/9$, $w_5 = w_6 = w_7 = w_8 = 1/36$, $c_s = c/\sqrt{3}$ is the sound speed of the model. The density and velocity can be directly evaluated by taking the zeroth and first moments of particle density distribution functions, respectively:

$$\rho = \sum_\alpha f_\alpha = \sum_\alpha f_\alpha^{eq} \tag{4}$$

$$\rho \boldsymbol{u} = \sum_\alpha \boldsymbol{e}_\alpha f_\alpha = \sum_\alpha \boldsymbol{e}_\alpha f_\alpha^{eq} \tag{5}$$

and the kinematic viscosity ν is determined by

$$\nu = (\tau - \frac{1}{2}) c_s^2 \Delta t \tag{6}$$

When external forces exist, such as gravity. In the lattice BGK model, it is known that the whole force could be added to the lattice Boltzmann equation as shown in the work by Guo et al (2002). In the present model, the boundary force can be evaluated by momentum exchange method.

Immersed boundary method

The interaction between the boundary grid-points and fluid nodes in the immersed boundary method can be expressed as

$$\boldsymbol{F}(\boldsymbol{x},t)=\int_{\Omega}\boldsymbol{g}(\boldsymbol{X},t)\delta(\boldsymbol{x}-\boldsymbol{X})ds \tag{7}$$

Where $\boldsymbol{F}(\boldsymbol{x},t)$ is the force generated by the boundaries onto the fluid, $\delta(\boldsymbol{x}-\boldsymbol{X})$ is the Dirac delta function, $\boldsymbol{X}(X,Y)$ is the coordinate of Lagrangian boundary points, $\boldsymbol{g}(\boldsymbol{X},t)$ is the Lagrangian force density.

The discretized form of Eq. (7) using a regularized discrete delta function D_{ij} are expressed as

$$\boldsymbol{F}(\boldsymbol{x}_{ij},t)=\sum_{l}\boldsymbol{g}(\boldsymbol{X}_l,t)D_{ij}(\boldsymbol{x}_{ij}-\boldsymbol{X}_l)\Delta s_l \tag{8}$$

where Δs_l is the arc length of the boundary element.

The discrete delta function D_{ij} appearing in Eq. (8) is a smoothed approximation to the Dirac delta function $\delta(\boldsymbol{x}-\boldsymbol{X})$. The detailed derivation procedures and several forms were presented by Peskin (2002). We apply the common form as follows:

$$D_{ij}(x_{ij}-X_l)=\frac{1}{h^2}\delta_h(\frac{x_{ij}-X_l}{h})\delta_h(\frac{y_{ij}-Y_l}{h}) \tag{9}$$

with

$$\delta_h(a)=\begin{cases}\frac{1}{4}(1+\cos(\frac{\pi a}{2})) & |a|\le 2\\ 0 & otherwise\end{cases} \tag{10}$$

where $h=\delta_x$ is the lattice spacing, the integral of function $\delta_h(a)$ is equal to one.

boundary force calculation

In the conventional immersed boundary-lattice Boltzmann method, the penalty method in previous work by Feng et al (2004) or the direct forcing method as shown in the work by Fadlun et al (2000) was proposed to calculate the boundary force $\boldsymbol{F}(\boldsymbol{X},t)$. Recently, a simple method for computing the boundary force was proposed as shown in the work by Niu et al (2006), in which the momentum exchange at the boundary is used to compute the force, however, the computed forces at the boundary have some oscillations. We use a novel momentum exchange at the boundary to compute the boundary force in the previous work by Wen (2013).

The force on the fluid-structure boundary can be written as

$$F(X_l,t)=\sum_{\alpha}\left[(e_{\alpha}-u)f_{\alpha}(x,t)-(e_{\beta}-u)f_{\beta}(x,t)\right] \tag{11}$$

Here $f_{\alpha}(x,t)$ indicates the mass component straming into the boundary and contributing a momentum increment $(e_{\alpha}-u)f_{\alpha}(x,t)$ to the boundary, while $f_{\beta}(x,t)$ streams out of the boundary and contributes a momentum decrement $(e_{\beta}-u)f_{\beta}(x,t)$.

The total boundary force F_T and the torque T_T exerted on the boundary points are expressed as

$$F_T(X_l,t)=\sum_{l}F(X_l,t)\Delta s_l \tag{12}$$

$$T_T=\sum_{l}(X_l-R)\times F(X_l,t)\Delta s_l \tag{13}$$

where R is the mass center of the structure, and the summation runs over all the fluid structure boundary.

Flows past a circular cylinder

In order to examine the accuracy and efficiency of the proposed immersed boundary-Lattice Boltzmann method (IB-LBM), numerical simulations of the viscous flow past a circular cylinder are carried out. The computational domain is set by $22D\times22D$, D is the diameter of circular cylinder, the free stream velocity u_{∞} is taken as 0.05, and the free stream density ρ_{∞} is set to be 1.0, The drag and lift coefficients are defined by

$$C_d=\frac{F_D}{0.5\rho u_{\infty}^2 D} \quad , \quad C_l=\frac{F_L}{0.5\rho u_{\infty}^2 D} \tag{14}$$

Where the drag force F_D and the lift force F_L on the immersed body are calculated as

$$F_D=-\left\{\frac{\mathrm{d}}{\mathrm{d}t}\left(\int_R u_1\mathrm{d}R\right)+\int_{\Omega}\left[u_1\boldsymbol{u}\bullet\boldsymbol{n}+pn_1-\mu\left(\frac{\partial u_1}{\partial x_i}+\frac{\partial u_i}{\partial x_1}\right)\right]\mathrm{ds}\right\} \tag{15}$$

$$F_L=-\left\{\frac{\mathrm{d}}{\mathrm{d}t}\left(\int_R u_2\mathrm{d}R\right)+\int_{\Omega}\left[u_2\boldsymbol{u}\bullet\boldsymbol{n}+pn_2-\mu\left(\frac{\partial u_2}{\partial x_i}+\frac{\partial u_i}{\partial x_2}\right)\right]\mathrm{ds}\right\} \tag{16}$$

Where $\boldsymbol{n}$ is the normal vector to the boundary of control surface, the subscripts 1 and 2 denote the *x*-direction and *y*-direction. Figure 1 shows the distribution of drag coefficients versus the Reynolds number in logarithmic scale. Obviously, the present results compare very well with the experimental data as shown in the work by Tritton (1959) and other numerical results by Niu et al (2006) and Wu et al (2009). Table 1 displays the comparison of non-dimensional length of recirculating eddy L/D, here L is the length of the recirculating region. Comparing the present method and other research work at $\mathrm{Re}=20$ and $\mathrm{Re}=40$, we can see that our numerical results are in good agreement with previous ones.

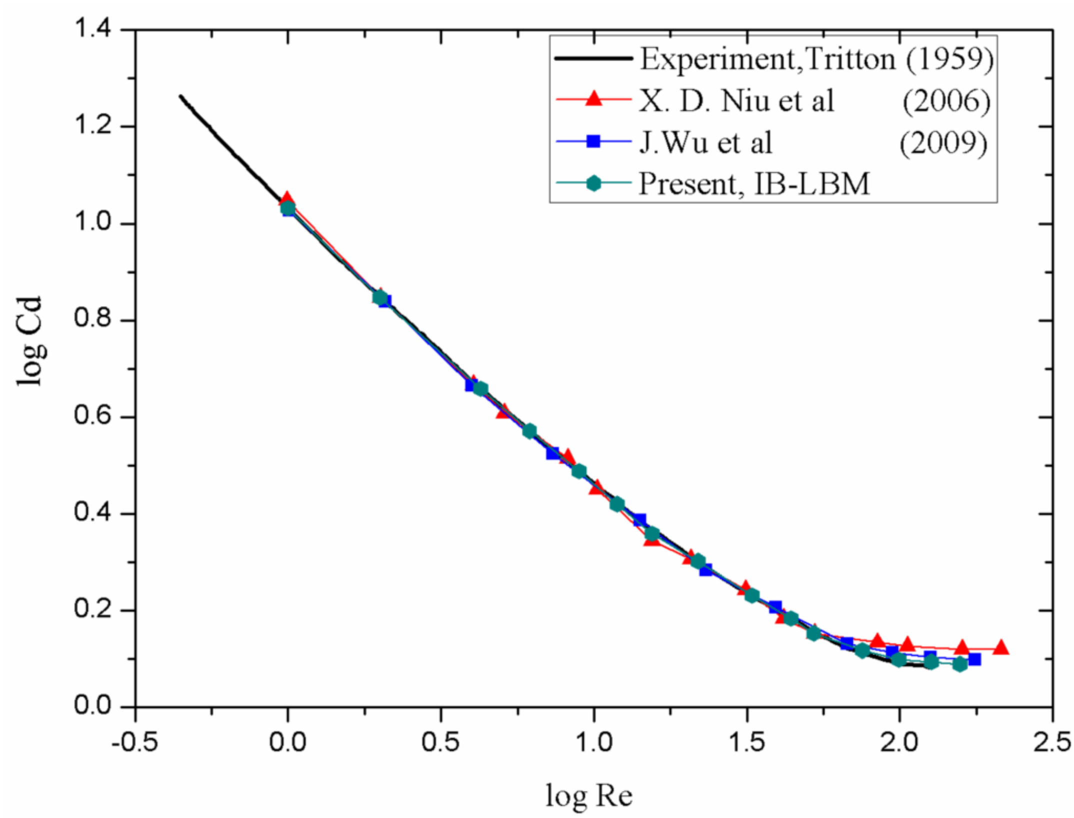

Figure 1. Comparison of drag coefficients

TABLE 1. Comparison of recirculating length (L/D) with previous studies

Authors	Re=20	Re=40
Dennis and Chang (1959)	0.94	2.345
Nieuwstadt and Keller (1973)	0.893	2.179
Coutanceau and Bouard (1977)	0.93	2.13
Fornberg (1980)	0.91	2.24
He and Doolen (1997)	0.921	2.245
Tseng and Ferziger (2003)	--	2.21
Shu and Liu (2007)	0.90	2.20
Present	0.92	2.26

Conclusions

we couple the immersed boundary method and lattice Boltzmann method, presenting a Galilean invariant momentum exchange equation by introducing the relative velocity into the interfacial momentum transfer to compute the boundary force. The present method preserves the merits of the LBM and the IBM by using two unrelated computational meshes, an Eulerian mesh for the flow domain and a Lagrangian mesh for the moving boundaries in the flow. The simplicity of the Eulerian mesh facilitates the numerical implementation of the LBM, and the generality of the Lagrangian mesh makes it easy to handle complex boundaries.

The present method is validated by its application to simulate the steady flows past a circular cylinder. The obtained results are in good agreement with available data in the literature.

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