# Error Analysis of Dynamical Measurement System Based on Binocular Vision

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## Abstract

With the rapid development of industrial cameras, binocular stereo vision is widely used in the field of manufacturing. Using two or more image points of one point in space to restore space depth information is called stereo vision, which process is three-dimensional reconstruction. Nowadays binocular vision which applied for vibration test is widely used .While binocular stereo vision is normally used in static test and has been rarely reported for the dynamical measurement. So it is important to establish this dynamic error model and analyze error sources to improve measurement accuracy. In this paper, an error analysis of binocular vision for dynamical test is presented. For this measurement system of binocular vision, errors of results are generated by the deviation between the calculated world coordinate values and the actual world coordinate values. On the assumption that the calibration result is correct, an error model of dynamic measurement is established in the article which the deviation of Z coordinate value between the ideal point and the reconstructed point is mainly studied. This deviation is so-called reconstruction error. In this model, two cameras are non-synchronized. And the effects of motion parameters, such as amplitude, frequency, phase when double cameras take photos of one point respectively with simple harmonic motion or sinusoidal movements are analyzed. The proposed model is close to the actual situation and can be used to set up cameras and control cameras' non-synchronized time to improve measurement accuracy when performing dynamic measurement.

Keywords: Binocular vision, Error model, Dynamic measurement, 3D reconstruction.

## Introduction

In the field of measurement, vibration test is widely studied. Now two common methods can be used to achieve vibration test. One can be achieved indirectly by sensors and the other method is to use a laser. The former requires the sheet sensor to attach on the target point, so the structure of this method is complicated with lower precision; although the latter method is simple, it can't be applied to long distance test and the intensity is susceptible to be affected by external environment. These two methods are single-point measurement method. Currently, visual technology is developing rapidly. And vibration test that achieved by the use of visual measurement technology has been proposed. Especially binocular vision technology is widely used. This emerging measurement method is a full field type which has various advantages, such as experimental convenience, high precision, quick access to large amounts of information, automatic processing <sup>[1]</sup>.

According to what are mentioned above, understanding and learning binocular vision technology has great significance. The basic model of binocular vision is pinhole model, which theoretically requires at least two photos to be realized 3D information of one scene point. This process is called 3D reconstruction <sup>[2]</sup>. Theory of binocular vision in practical application has a high practical value, for example, in the industrial field binocular vision technology can be used to identify and positioning some parts of the shape of quite different shape and colors and make them separated; in the field of public security, binocular vision can be used to reproduce the scene of the accident scene and determine responsibility <sup>[3-4]</sup>. In view of that, binocular vision has gradually improved and

affects our lives. In order to improve the measurement accuracy, the establishment and analysis of 3D reconstruction error model based on binocular vision is very important. And error models based on binocular vision are involved in many papers. H.J.Yu analyzed the relationship between structural parameters of binocular vision and measurement results. Theoretically, the relationship between coordinate measurement precision and angle of two cameras' optical axis or the baseline distance are analyzed <sup>[5]</sup>. In order to improve the measurement precision for visual measure system, research on the influence of stereovision structure based on binocular vision is presented by A.X.Guo. A structure parameter model of binocular visual measure system is investigated with the trigonometric method <sup>[6].</sup> Q.j.Li once proposed a measurement system of parallel-axes binocular stereoscopic. The Monte-Carlo method was used in simulating and analyzing the errors which caused by the calibrated parameters <sup>[7]</sup>. L.F.Cai analyzed image recognition errors' influence on the measurement accuracy of the visual system. And the relationship between structural parameters of visual system and measurement errors is introduced <sup>[8]</sup>. S.F performed left-right consistency checks and compared matching error between the corresponding pixels in binocular disparity calculation, and classified the stereoscopic images into non-corresponding, binocular fusion, and binocular suppression regions <sup>[9]</sup>. Gal'Pem derived some basic relations analytically for the determination of the degree of distortion and the connection between coordinates of object and image for stereoscopic vision are derived. The degree of distortion is calculated and its effect on stereo-vision is discussed <sup>[10]</sup>. Derek Bradley proposed an error correction algorithm which can solve the mismatch problem of multi-cameras' polar constraint. And this error correction method error algorithm is more effective in reducing camera calibration errors in the same scene than error algorithm of reprojection<sup>[11]</sup>.

In the previous paragraph, error models mostly belong to model of static measurement. This static theory has been quite mature because the measurement error in this stationary case is easier to control. While error theory of dynamic test based on binocular vision is very rare. Because this dynamic measurement should consider not only the factors that must be considered in static measurement, such as calibration error, matching error, pixel positioning error of camera's CCD, etc, but also asynchronous error and the movement of moving objects. X.L.Zhang built a system of binocular vision measurement which the two optic axes are parallel. That was adopted to measure the attitude of measured missile which is moving <sup>[12]</sup>. This model of binocular vision is not realistic and does not take into account the motion form of measured object that has effect on the measurement results. The motion parameters such as amplitude, frequency and phase will have a certain impact on measurement accuracy. Today achieving vibration test by binocular vision is an irresistible trend which belongs to the category of dynamic test. Therefore, the establishment of error model based on binocular vision in dynamic measurement and reconstructed error analysis on various cameras' parameters and motion parameters of measured object is the key.

In this paper, a dynamic measurement model base on binocular vision which relatively closes to the reality is mainly established. The double cameras has a certain distance between front and rear, left and right, up and down and there is a tiny angle between cameras' optical axes. This article focuses on the analysis on the deviation of 3D coordinate values that caused by these two non-synchronized cameras between original point and reconstructed point under the premise there is no calibration error. First, the deviation is calculated when a spatial point does uniform motion respectively on each axis based on that established model. Second, assuming that the measured point respectively does simple harmonic motion on each axis or propagates forward as a form of a sine wave on a plane, the error equals to precious deviation which affected by motion parameters is discussed.

#### **One Dynamic Error Model Based on Binocular Vision**

For actual measurement, one system of binocular visual is not a standard system, for example, two cameras has a certain distance between front and rear, left and right, up and down, and two optical axles of cameras are not necessarily parallel. Theoretically parameters of camera system will affect the measurement accuracy such as focal length, the baseline distance of two cameras, the angle between the two optical axles of cameras and so on. In order to close to the actual situation and consider these parameters, a model based on binocular vision as shown below is built.

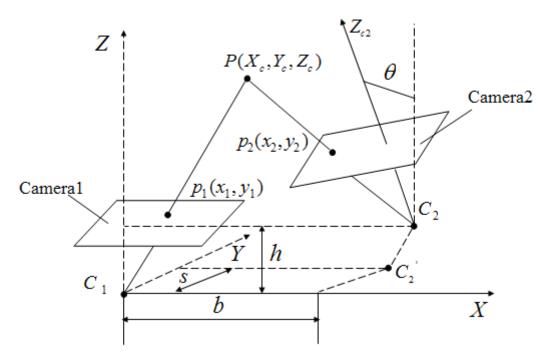


Figure 1. A model of 3D reconstruction based on binocular vision

The system parameters are reflected in fig1 and  $C_1$ ,  $C_2$  are the optical centers of the two cameras. The right optical axis revolved around Z-axis which has an extremely angle  $\theta$  between the two optical axles. This system puts the coordinate system of the left camera as the world coordinate system. Spatial point P has two projection points on the two cameras, respectively  $p_1(x_1, y_1)$  and  $p_2(x_2, y_2)$ . And some formulas can be obtained by the principle of similar triangle.

$$x_{1} = f \frac{X_{c}}{Z_{c}} \quad y_{1} = f \frac{Y_{c}}{Z_{c}} \quad x_{2} = f \frac{X_{c}}{Z_{c}} \quad y_{2} = f \frac{Y_{c}}{Z_{c}}$$
(1)

Coordinate systems of the two cameras can be interchangeable, so

$$\begin{bmatrix} \cos\theta & \sin\theta & 0 & b \\ -\sin\theta & \cos\theta & 0 & s \\ 0 & 0 & 1 & h \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix} = \begin{bmatrix} X_c' \\ Y_c' \\ Z_c' \\ 1 \end{bmatrix}$$
(2)

Then coordinate values of point  $P(X_c, Y_c, Z_c)$  can be obtained according to (1) (2)

$$X_{c} = \frac{x_{1}Z_{c}}{f} \qquad \qquad Y_{c} = \frac{y_{1}Z_{c}}{f}$$

$$Z_{c} = \frac{(x_{2} - y_{2})h - f(b - s)}{(\cos\theta + \sin\theta)x_{1} + (\sin\theta - \cos\theta)y_{1} - (x_{2} - y_{2})} \approx \frac{(x_{2} - y_{2})h - f(b - s)}{(x_{1} - y_{1}) + \theta(x_{1} + y_{1}) - (x_{2} - y_{2})}$$
(3)

The above formulas are obtained in the static case which reflects the 3D information of one spatial point by the use of binocular vision. While theory of dynamic testing based on binocular vision is not yet ripe. So the following dynamic system in fig.2 based on binocular vision is established which the double cameras are non-synchronized in order to analyze the deviation of the 3D coordinate values between actual reconstructed point P and original point P.

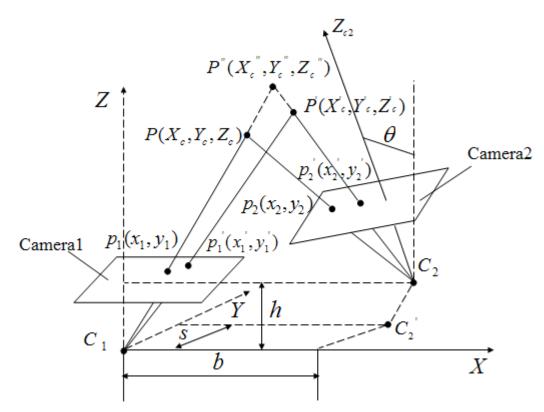


Figure2. Schematic of 3D reconstruction in dynamic measurement

The schematic of dynamic test is shown in figure2. The left camera exposures earlier than the right one and point P moves to point P when the right camera starts to take pictures. Point P also has the two projection points in imaging planes, respectively  $p_1(x_1, y_1)$ ,  $p_2(x_2, y_2)$ . In the actual process of reconstruction, projection point  $p_1$  and  $p_2$  are used to restore 3D coordinate information.

Here assuming that the point *P* does uniform linear motion respectively along X-axis, Y-axis, Z-axis and corresponding displacement are  $\Delta X$ ,  $\Delta Y$ ,  $\Delta Z$ . And the deviation between *P* and *P*<sup>"</sup> will be calculated in the latter.

1) When Point P does uniform motion along X-axis, reconstructed 3D coordinate values are

$$X_{c}^{"} = \frac{x_{1}Z_{c}^{"}}{f}$$
  $Y_{c}^{"} = \frac{y_{1}Z_{c}^{"}}{f}$ 

$$Z_{c}^{"} = \frac{(x_{2}^{'} - y_{2}^{'})h - f(b - s)}{(x_{1} - y_{1}^{'}) + \theta(x_{1} + y_{1}^{'}) - (x_{2}^{'} - y_{2}^{'})} = \frac{(X_{c} + \Delta X)(1 + \theta)h + h(\theta - 1)Y_{c} - (b - s)Z_{c}}{Z_{c}(1 + \theta)(h - \Delta X) + (\theta - 1)Y_{c}h - (b - s)Z_{c}}Z_{c}$$
(4)

$$\Delta Z_{c} = Z_{c}^{"} - Z_{c} = \frac{(1+\theta)(Z_{c}+h)\Delta X}{Z_{c}(1+\theta)(h-\Delta X) + (\theta-1)Y_{c}h - (b-s)Z_{c}}Z_{c}$$
(5)

2) Similarly when Point P does uniform motion along Y-axis, reconstructed 3D coordinate values are

$$X_{c}^{"} = \frac{x_{1}Z_{c}^{"}}{f} \quad Y_{c}^{"} = \frac{y_{1}Z_{c}^{"}}{f} \quad Z_{c}^{"} = \frac{hX_{c}(1+\theta) + h(\theta-1)(Y_{c}+\Delta Y) - (b-s)Z_{c}}{X_{c}(1+\theta)h + (\theta-1)(Y_{c}h - \Delta YZ_{c}) - (b-s)Z_{c}}Z_{c}$$
(6)

$$\Delta Z_{c} = Z_{c}^{"} - Z_{c} = \frac{(\theta - 1)(h + Z_{c})\Delta Y}{X_{c}(1 + \theta)h + (\theta - 1)(Y_{c}h - \Delta YZ_{c}) - (b - s)Z_{c}}Z_{c}$$
(7)

3) When Point P does uniform motion along Z-axis, reconstructed 3D coordinate values are

$$X_{c}^{"} = \frac{x_{1}Z_{c}^{"}}{f} \quad Y_{c}^{"} = \frac{y_{1}Z_{c}^{"}}{f} \quad Z_{c}^{"} = \frac{hX_{c}(1+\theta) + h(\theta-1)Y_{c} - (b-s)(Z_{c} + \Delta Z)}{(1+\theta)X_{c}(\Delta Z + h) + (\theta-1)(\Delta Z + h)Y_{c} - (b-s)Z_{c}}Z_{c}$$
(8)

$$\Delta Z_{c} = Z_{c}^{"} - Z_{c} = \frac{[s - b + Y_{c} - X_{c} - \theta(X_{c} + Y_{c})]\Delta Z}{(1 + \theta)X_{c}(\Delta Z + h) + (\theta - 1)(\Delta Z + h)Y_{c} - (b - s)Z_{c}}Z_{c}$$
(9)

4) Assuming that point *P* moves to  $P'(X_c + \Delta X, Y_c + \Delta Y, Z_c + \Delta Z)$  when the right camera starts to take pictures, the actual reconstructed 3D coordinate values are obtained as follows.

$$X_{c}^{"} = \frac{x_{1}Z_{c}^{"}}{f} \qquad Y_{c}^{"} = \frac{y_{1}Z_{c}^{"}}{f} Z_{c}^{"} = \frac{\left[(X_{c} + \Delta X)(1 + \theta) + (\theta - 1)(Y_{c} + \Delta Y)\right]h - (Z_{c} + \Delta Z)(b - s)}{(1 + \theta)(X_{c}h + X_{c}\Delta Z - Z_{c}\Delta X) + (\theta - 1)(hY_{c} + Y_{c}\Delta Z - Z_{c}\Delta Y) - (b - s)Z_{c}} Z_{c}$$
(10)

$$\Delta Z_{c} = Z_{c}^{*} - Z_{c} = \frac{(1+\theta)(h\Delta X - X_{c}\Delta Z + Z_{c}\Delta X) + (\theta - 1)(h\Delta Y - Y_{c}\Delta Z + Z_{c}\Delta Y) - (b - s)\Delta Z}{(1+\theta)(X_{c}h + X_{c}\Delta Z - Z_{c}\Delta X) + (\theta - 1)(hY_{c} + Y_{c}\Delta Z - Z_{c}\Delta Y) - (b - s)Z_{c}} Z_{c}$$
(11)

#### **Error Analysis of Dynamic Measurement**

For this dynamic testing model based on binocular vision in fig.2, point P can moves with different ways, such as harmonic motion, sinusoidal motion, etc. The motion parameters, such as amplitude, frequency and phase, of point P which has effect on 3D coordinate values will be analyzed when point P respectively does these two motions.

#### A. Error Analysis of Z Coordinate of Point P with Harmonic Motion

In the following, point *P* does harmonic motion on each axis. Fig.3 reflects characteristics of harmonic motion which  $P_t = A\sin(2\pi f t + \varphi)$ . The area of shaded portion represents the distance  $\Delta S$  that point *P* has traveled during the unsynchronized time  $\Delta t$  of the two cameras.

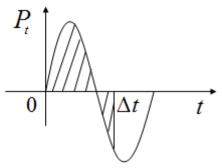


Figure3. Characteristics of harmonic motion

$$\Delta S = \int_0^{\Delta t} \left| \frac{dx}{dt} \right| dt = \int_0^{\Delta t} 2\pi A f \left| \cos(2\pi f t + \varphi) \right| dt \tag{12}$$

1) When point *P* does harmonic motion on X-axis, the distance  $\Delta X$  which point *P* has traveled on X-axis is equal to  $\Delta S$ . Put  $\Delta S$  into the above equation (5), then the error of Z coordinate between origin point *P* and reconstructed point *P*<sup>"</sup> in this occasion can be got. In order to discuss the measurement error here, supposing that  $\theta = 5^{\circ}$ ,  $X_c = 12 \ mm$ ,  $Y_c = 10 \ mm$ ,  $Z_c = 10 \ mm$ ,  $b = 50 \ mm$ ,  $s = h = 10 \ mm$ ,  $t_0 = 0s$ ,  $\Delta t = 5 \ us$ . Look at figure 4(a), if given  $f = 50 \ Hz$ ,  $\varphi = \frac{\pi}{6}$ , the relationship between error and amplitude can be obtained. And the

greater amplitude, the greater error; if given A = 10mm,  $\varphi = \frac{\pi}{6}$ , the relationship in figure 4(b) between error and frequency can be obtained. And the higher frequency, the greater error; if given A = 10mm, f = 50Hz, the relationship in figure 4(c) between error and phase can be obtained, the relationship curve is a cycle cure with a period of  $\pi$ . According to this picture, phase has little effect on the error.

2) When point P does harmonic motion on Y-axis, the distance  $\Delta Y$  which point P has traveled on Y-axis also equals to  $\Delta S$ . If  $\Delta S$  was put into the above equation (7), the error of Z coordinate can also be obtained. Error analysis of Z coordinate is similar to the upper part. Here given the same parameter values like the upper part, respectively relation formulas between error and amplitude or frequency or phase can be obtained. The following cures reflect the relationships between the error which caused by Y-axis and these three parameters.

3) When point P does harmonic motion on Z-axis, the distance  $\Delta Z$  that point P has traveled on Z-axis also equals to  $\Delta S$ . If  $\Delta S$  was put into the above equation (9), the error of Z coordinate can be obtained. The process of error analysis is the same as the previous two cases. And the relationships between error and various parameters are shown below in figure 4.

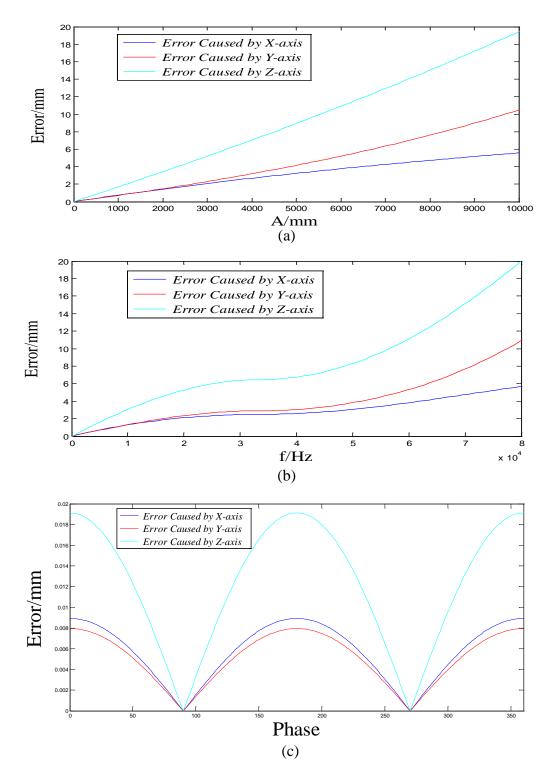


Figure4.The relationships between motion parameters and errors of Z coordinate of point *P* with harmonic motion on each axis (a) Relationship between error and amplitude. (b) Relationship between error and frequency. (c) Relationship between error and phase.

Figure4 (a), (b), (c) respectively reflects the relationships between error of Z coordinate and amplitude, frequency or phase when point P does harmonic motion on the each axis. Error increases with the larger amplitude or greater frequency. And when frequency varies in the range of

(14)

30 - 40 KHz, the error stays in a flat stage. At last, the relation curves between error and phase are cycle curves with a period of  $\pi$  which reflect phase has little effect on the error.

## B. Error Analysis of Z Coordinate of Point P with Sinusoidal Motion

Here the second motion of point P which point P propagates forward as a sine wave on XOY plane will be discussed. The characteristics of sinusoidal motion are reflected below in fig.5. And this wave equation is formula (13). Assuming velocity v is one constant and equals to  $3.4 \times 10^4 \text{ mm/s}$ , then displacements including  $\Delta X$  and  $\Delta Y$  respectively on the horizontal direction and the vertical direction can be calculated when the point P moves to P' during the time  $\Delta t$  in this case.

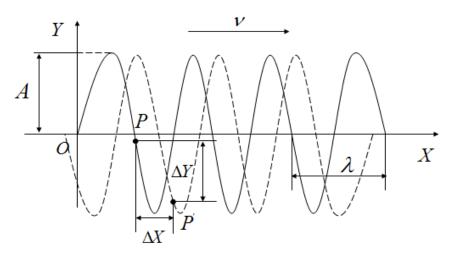


Figure 5. Characteristics of sinusoidal motion

$$y = A\cos[2\pi f(t - \frac{x}{\nu}) + \varphi]$$
(13)

$$\Delta X = \nu \Delta t$$

$$\Delta Y = 4 \left\| \Delta t f \right\| A + A \cos[2\pi f(t_0 + \Delta t - \frac{X_c + \Delta X}{\nu}) + \varphi] - \cos[2\pi f(t_0 - \frac{X_c}{\nu}) + \varphi]$$
(15)

Here  $\theta = 5^{\circ}$ ,  $X_c = 12 \ mm$ ,  $Y_c = 10 \ mm$ ,  $Z_c = 10 \ mm$ ,  $b = 50 \ mm$ ,  $s = h = 10 \ mm$ ,  $t_0 = 0s$  and substituting  $\Delta X$ ,  $\Delta Y$ ,  $\Delta Z = 0$  into equation(11), then the relationship between error and amplitude, phase, wavelength or frequency can be obtained. Figs of the results will be given.

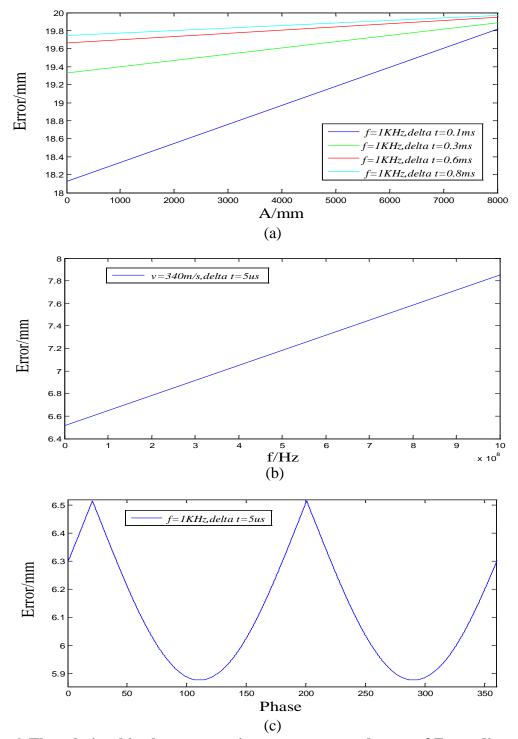


Figure 6. The relationships between motion parameters and error of Z coordinate of point P with sinusoidal motion on XOY plane. (a) Relationship between error and amplitude. (b). Relationship between error and frequency. (c) Relationship between error and phase.

Figure 4 (a) is the curve which reflects the relationship between frequency and error of Z coordinate. When the frequency of this sine wave is a constant which f = 1KHz, the relationships between the amplitude and error of Z coordinate are analyzed when to take special value of the non-synchronization time  $\Delta t$  in the range of 0-1/4 T, 1/4 T -2/4T, 2/4T- 3/4T and 3/4T-4/4T. From the four curves in fig.4 (a), this error increases with the increasing amplitude. And every error curve

does not pass through the origin point, because the error will be produced which results in a lateral displacement on X-axis because of velocity, through amplitude is zero. Comparing these four curves in figure 4(a), the greater the asynchronous time  $\Delta t$ , the larger the error of Z coordinate. Figure 4 (b) reflects the relationship between this error and frequency. When velocity is certain and a value of frequency is determined, then the wavelength can be obtained. In this case, the error increases as the frequency increases. This error curve involved with frequency does not pass through the origin point either due to the vibration on longitudinal direction which produces the displacement on Y direction. The diagram (c) is a cycle curve with a period of  $\pi$  and this error influence on the phase can not be neglected.

## Conclusions

In this paper, a dynamic error model based on binocular vision is mainly established. This model is not a standard system. Specific camera settings are reflected in the above fig1. The deviation of Zcoordinate between the origin point P and the reconstructed point is analyzed when two cameras are not synchronized and the spatial point P respectively does uniform motion, harmonic motion on each axis or sinusoidal motion on a plane. When point P does uniform motion on each axis, this deviation can be calculated. When point P does harmonic motion on each axis, that deviation of results that affected by the motion parameters are analyzed. Error equals to that deviation increases with the greater amplitude or larger frequency. The relationship between phase and error is a cycle curve which has litter effect on accuracy. When point P propagates forward as a form of sine wave on XOY plane which velocity is a constant, firstly the greater frequency, then the smaller wavelength and greater error; secondly obviously error increases with the increasing amplitude; thirdly the curve involved with phase and error is also a periodic curve and the effect caused by phase can not be neglected. Thus, for the dynamic measurement systems, not only the structure of binocular vision system, but also the movement pattern and parameters of one target point will have impact on measurement accuracy. This proposed model can be helpful to set up cameras and can reduce measurement error caused by asynchronous cameras when it comes to dynamic vibration test based on binocular vision.

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