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Hybrid probabilistic interval dynamic analysis of vehicle-bridge interaction system with uncertainties

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Abstract

Hybrid probabilistic interval dynamic analysis of vehicle-bridge interaction system with a mixture of random and interval properties is studied in this paper. The vehicle's parameters are considered as interval variables and the bridge's parameters are treated as random variables. By introducing the random interval moment method into the dynamic analysis of vehicle-bride interaction system, the expressions for the mean value and standard deviation of the random interval bridge dynamic response are developed. Examples are used to illustrate the effectiveness of the presented method. A hybrid simulation method combining direct simulations for interval variables and Monte-Carlo simulations for random variables is implemented to validate the computational results.

Keywords: Vehicle-bridge interaction system, probabilistic interval analysis, random interval moment method, random interval dynamic response.

Introduction

The coupled vehicle-bridge dynamic system has attracted considerable attentions over the past two decades (Yang and Lin, 2005; Ju and Lin, 2007; Zhang et al., 2008). The values of system parameters are given precisely in most of studies. Actually, vehicles moving on a bridge have nondeterministic characteristics because the system parameters are not constant.

Probabilistic methods are preferred when information of uncertain parameters in the form of preference probability function is provided. And these have been widely used to predict the response and in the implementation of structural system reliability evaluation of uncertainty (Liu et al., 2011). In probabilistic methods, uncertain parameters are modeled as random variables/fields and uncertainties of loads are described by random processes/variables. However, sometimes it is hard to get the enough probabilistic information for structural parameters as their values are affected by a lot of non-deterministic factors. Meanwhile, loads of many scenarios can hardly be modeled as random variables due to large changes in their magnitudes. The interval methods can be used when the probability function is unavailable but the range of the uncertain parameter is known. In the past decade, significant progress in analysis and optimal design of structures with bounded parameters has been achieved (Qiu et al., 2009; Jiang et al., 2008; Impollonia and Muscolino, 2011).

It is desirable to model structural parameters/loads as random variables if sufficient information can be obtained to form the probability density functions. Meanwhile, some structural parameters/loads might be best considered as interval variables if the information/data are not enough to model uncertain structural parameters and loadings as random variables, especially in the early design stages. Consequently, hybrid probabilistic interval analysis and reliability assessment of structures with a mixture of random and interval properties has been conducted (Gao, 2010). The random interval moment method has been developed by the authors to determine the mean value and standard deviation of random interval responses of structures under static forces (Gao, 2010).

As aforementioned, some parameters of vehicle-bridge interaction system could be considered as random variables and some of them might be assumed as interval variables. For example, the change range of vehicle's mass is large due to the different loading conditions; therefore, these can be taken as interval variables. In contrast, the change ranges of bridge's parameters are small because of the strict manufacturing standards, which can be considered as random variables. Therefore, a hybrid probabilistic interval analysis model for vehicle-bridge coupled systems needs to be developed.

Random interval moment method

Let $X(R)$ be the set of all real random variables on a probability space (Ω, A, P), x^R is a random variable of $\mathrm{X}(R)$. R denotes the set of all real numbers. μ_x (or $\bar{x}$) and σ_x are the mean (deterministic) value and standard deviation of x^R, respectively. $y^I = [\underline{y}, \overline{y}] = \left\{ t, \underline{y} \le t \le \overline{y} \middle| \underline{y}, \overline{y} \in R \right\}$ is an interval variable of $I(R)$ which denotes the set of all the closed real intervals. $\underline{y}$ and $\overline{y}$ are the lower and upper bounds of interval variable y^I, respectively. Interval variable y^I can also be written as

$$y^I = y^c + \Delta y^I \text{ ; } \Delta y^I = [-\Delta y, +\Delta y] \text{ ; } y^c = \frac{\underline{y} + \overline{y}}{2} \text{ ; } \Delta y = \frac{\overline{y} - \underline{y}}{2} \quad \Delta y_F = \frac{\Delta y}{y^c} \tag{1}$$

where y^c, Δy , Δy^I and Δy_F represent the midpoint value, maximum width (interval width), uncertain interval and interval change ratio of the interval variable y^I .

Without loss of generality, random interval variable Z^{RI} is the function of multiple random and interval variables, which are respectively represented by random vector $\vec{X}^R = (x_1^R, x_2^R, \cdots, x_n^R)$ and interval vector $\vec{Y}^I = (y_1^I, y_2^I, \cdots, y_m^I)$. The deterministic values of $\vec{X}^R$ and $\vec{Y}^I$ are $\bar{\vec{X}} = (\bar{x}_1, \bar{x}_2, \cdots, \bar{x}_n)$ and $\vec{Y}^c = (y_1^c, y_2^c, \cdots, y_m^c)$.

The Taylor series to the first-order of the random interval variable $Z^{RI} = f(\vec{X}^R, \vec{Y}^I)$ about ($\vec{X}^R, \vec{Y}^I$) is expressed as

$$Z^{RI} = f(\vec{X}^R, \vec{Y}^I) = f(\bar{\vec{X}}, \vec{Y}^I) + \sum_{i=1}^{n} \left\{ \left. \frac{\partial f}{\partial x_i^R} \right|_{\bar{\vec{X}}, \vec{Y}^I} \right\} \cdot (x_i^R - \bar{x}_i) + R$$

$$= f(\bar{\vec{X}},\vec{Y}^c)+\sum_{j=1}^{m}\left\{\left.\frac{\partial f}{\partial y_j^I}\right|_{\bar{\vec{X}},\vec{Y}^c}\right\}\cdot\Delta y_j^I$$

$$+\sum_{i=1}^{n}\left\{\left.\frac{\partial f}{\partial x_i^R}\right|_{\bar{\vec{X}},\vec{Y}^c}+\sum_{j=1}^{m}\left\{\left.\frac{\partial^2 f}{\partial x_i^R\partial y_j^I}\right|_{\bar{\vec{X}},\vec{Y}^c}\right\}\cdot\Delta y_j^I\right\}\cdot(x_i^R-\bar{x}_i)+R \qquad (2)$$

where R is the remainder term.

From this equation, and the higher order terms R is ignored, the expectation and variance of random interval variables $Z^{RI}=f(\vec{X}^R,\vec{Y}^I)$ can be calculated as (Gao et al., 2010)

$$\mu_{Z^{RI}}=E(Z^{RI})=f(\bar{\vec{X}},\vec{Y}^c)+\sum_{j=1}^{m}\{\left.\frac{\partial f}{\partial y_j^I}\right|_{\bar{\vec{X}},\vec{Y}^c}\}\Delta y_j^I \qquad (3)$$

$$\sigma^2_{Z^{RI}}=E\left(Z^{RI}-E(Z^{RI})\right)^2=\sum_{i=1}^{n}\sum_{k=1}^{n}\left\{\left.\frac{\partial f}{\partial x_i^R}\right|_{\bar{\vec{X}},\vec{Y}^c}+\sum_{j=1}^{m}\left\{\left.\frac{\partial^2 f}{\partial x_i^R\partial y_j^I}\right|_{\bar{\vec{X}},\vec{Y}^c}\right\}\Delta y_j^I\right\}$$

$$\cdot\left\{\left.\frac{\partial f}{\partial x_k^R}\right|_{\bar{\vec{X}},\vec{Y}^c}+\sum_{j=1}^{m}\left\{\left.\frac{\partial^2 f}{\partial x_k^R\partial y_j^I}\right|_{\bar{\vec{X}},\vec{Y}^c}\right\}\Delta y_j^I\right\}Cov\,(x_i^R,x_k^R)$$

$$=\sum_{i=1}^{n}\left\{\left.\frac{\partial f}{\partial x_i^R}\right|_{\bar{\vec{X}},\vec{Y}^c}+\sum_{j=1}^{m}\left\{\left.\frac{\partial^2 f}{\partial x_i^R\partial y_j^I}\right|_{\bar{\vec{X}},\vec{Y}^c}\right\}\Delta y_j^I\right\}^2\cdot Var\left(x_i^R\right)$$

$$+\sum_{i(\neq k)=1}^{n}\sum_{k=1}^{n}\left\{\left.\frac{\partial f}{\partial x_i^R}\right|_{\bar{\vec{X}},\vec{Y}^c}+\sum_{j=1}^{m}\left\{\left.\frac{\partial^2 f}{\partial x_i^R\partial y_j^I}\right|_{\bar{\vec{X}},\vec{Y}^c}\right\}\Delta y_j^I\right\}\cdot\left\{\left.\frac{\partial f}{\partial x_k^R}\right|_{\bar{\vec{X}},\vec{Y}^c}+\sum_{j=1}^{m}\left\{\left.\frac{\partial^2 f}{\partial x_k^R\partial y_j^I}\right|_{\bar{\vec{X}},\vec{Y}^c}\right\}\Delta y_j^I\right\}\cdot Cov\left(x_i^R,x_k^R\right) \qquad (4)$$

Vehicle-bridge interaction model

In the vehicle-bridge interaction system, the bridge is modeled as a simply supported beam (Yang, 2005) and the vehicle is represented by a half-car model as shown in Figure 1. Here, m_v, m_1 and m_2 denote the sprung and unsprung masses respectively; the suspension system is represented by two linear springs of stiffness k_{s1}, k_{s2} and two linear dampers with damping rates C_{s1}, C_{s2}; the tires are also modeled by two linear springs of stiffness k_{t1}, k_{t2} and two linear dampers with damping rates C_{t1}, C_{t2}; ρ, E, I and L are the mass per unit length, elastic modulus, moment of inertia and length of the beam respectively.

In this study, parameters of the vehicle m_1^I, m_2^I, and m_v^I, are considered as interval variables, meanwhile, bridge's parameters, ρ^R, E^R and I^R, are treated as random variables. The equation of motion governing the transverse vibration of the bridge under the moving vehicle with uncertain parameters can be written as

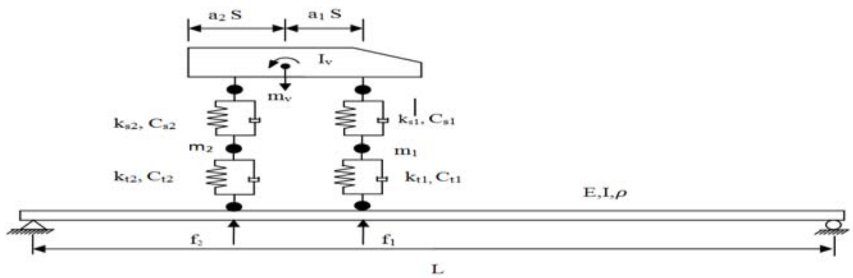


Figure 1. Model of vehicle-bridge interaction system

$$\rho^R \frac{\partial^2 W^{RI}(x,t)}{\partial t^2} + C\frac{\partial W^{RI}(x,t)}{\partial t} + E^R I^R \frac{\partial^4 W^{RI}(x,t)}{\partial x^4} = (f_1^{RI}(x,t) + f_2^{RI}(x,t))\delta(x-vt) \tag{5}$$

$$\begin{cases} f_1^{RI}(x,t) = -(m_1^I + a_2 m_v^I)g - k_{t1}^R(x_v(t)^{RI} - W^{RI}(x,t)\big|_{x=vt}) - C_{t1}^R(\dot{x}_v(t) - \dot{W}^{RI}(x,t)\big|_{x=vt}) \\ f_2^{RI}(x,t) = -(m_2^I + a_1 m_v^I)g - k_{t2}^R(x_v(t)^{RI} - W^{RI}(x,t)\big|_{x=vt}) - C_{t2}^R(\dot{x}_v(t) - \dot{W}^{RI}(x,t)\big|_{x=vt}) \end{cases} \tag{6}$$

where C is the damping of the bridge, $W^{IR}(x,t)$ is the random interval vertical displacement of the bridge, $x_v(t)^{RI}$ is the random interval vertical displacement of the moving vehicle, $f_1^{RI}(x,t)$ and $f_2^{RI}(x,t)$ are the random interval contact forces, $\delta(x-vt)$ is the Dirac delta function evaluated at the contact point at position $x = vt$, and v is the speed of the moving vehicle. Using the modal superposition method, the solution to Eq. (5) can be expressed as in terms of the mode shapes $\varphi_j(x)$.

In this paper, the Wilson's damping hypothesis is adopted. As vehicle mass is much less than the bridge mass and the tires' damping is quite small. Using the Duhamel integral solution, the displacement response of the bridge can be calculated by

$$W^{RI}(x,t) = -\sum_{j=1}^{\infty} \sin\frac{j\pi x}{L} \cdot \frac{2(m_1^I + m_2^I + m_v^I)g}{L\sqrt{1-\zeta_{bj}^2}\,\frac{j^2\pi^2}{L^2}\sqrt{\rho^R E^R I^R}} \cdot \int_0^t e^{-\zeta_{bj}\frac{j^2\pi^2}{L^2}\sqrt{\frac{E^R I^R}{\rho^R}}(t-\tau)} \sin\left(\sqrt{1-\zeta_{bj}^2}\,\frac{j^2\pi^2}{L^2}\sqrt{\frac{E^R I^R}{\rho^R}}(t-\tau)\right) \sin\frac{j\pi v t}{L}\, d\tau \tag{7}$$

In this study, the contribution of tires' stiffness to bridge vertical displacement response is omitted due to the assumption that the bridge mass is much greater than that of the vehicle(Yang, 2005). Additionally, bridge damping is treated as deterministic because the existing research outcomes show that the mechanism of structural damping is still not clear enough.

Furthermore, the lower and upper bounds of the mean value of the bridge's displacement $\mu(W^{RI})$ are given by

$$\underline{\mu\left(W^{RI}(x,t)\right)} = \frac{-2}{S_1\sqrt{\bar{\rho}\overline{EI}}}\sum_{j=1}^{\infty}\sin\frac{j\pi x}{L}\cdot\int_0^t\left((m_1^c+m_2^c+m_v^c)g\right)\cdot\bar{S}_2\sin\left(\bar{S}_3\right)\sin\frac{j\pi v\tau}{L}d\tau$$
$$-\left|\frac{-2g}{S_1\sqrt{\bar{\rho}\overline{EI}}}\sum_{j=1}^{\infty}\sin\frac{j\pi x}{L}\cdot\int_0^t\bar{S}_2\sin\left(\bar{S}_3\right)\sin\frac{j\pi v\tau}{L}d\tau\cdot\left(\Delta m_1+\Delta m_2+\Delta m_v\right)\right| \tag{8}$$

$$\overline{\mu\left(W^{RI}(x,t)\right)} = \frac{-2}{S_1\sqrt{\bar{\rho}\overline{EI}}}\sum_{j=1}^{\infty}\sin\frac{j\pi x}{L}\cdot\int_0^t\left((m_1^c+m_2^c+m_v^c)g\right)\cdot\bar{S}_2\sin\left(\bar{S}_3\right)\sin\frac{j\pi v\tau}{L}d\tau$$
$$+\left|\frac{-2g}{S_1\sqrt{\bar{\rho}\overline{EI}}}\sum_{j=1}^{\infty}\sin\frac{j\pi x}{L}\cdot\int_0^t\bar{S}_2\sin\left(\bar{S}_3\right)\sin\frac{j\pi v\tau}{L}d\tau\cdot\left(\Delta m_1+\Delta m_2+\Delta m_v\right)\right| \tag{9}$$

The lower and upper bounds of the variance of the bridge's displacements $\sigma^2\left(W^{RI}(x,t)\right)$ are

$$\underline{\sigma^2\left(W^{RI}(x,t)\right)} = \left\{\frac{\partial W^{RI}(x,t)}{\partial E^R} - \left(\frac{\partial^2 W^{RI}(x,t)}{\partial E^R\partial m_1^I}\cdot\Delta m_1 + \frac{\partial^2 W^{RI}(x,t)}{\partial E^R\partial m_2^I}\cdot\Delta m_2 + \frac{\partial^2 W^{RI}(x,t)}{\partial E^R\partial m_v^I}\cdot\Delta m_v\right)\right\}^2\cdot\sigma_E^2$$
$$+\left\{\frac{\partial W^{RI}(x,t)}{\partial I^R} - \left(\frac{\partial^2 W^{RI}(x,t)}{\partial I^R\partial m_1^I}\cdot\Delta m_1 + \frac{\partial^2 W^{RI}(x,t)}{\partial I^R\partial m_2^I}\cdot\Delta m_2 + \frac{\partial^2 W^{RI}(x,t)}{\partial I^R\partial m_v^I}\cdot\Delta m_v\right)\right\}^2\cdot\sigma_I^2$$
$$+\left\{\frac{\partial W^{RI}(x,t)}{\partial \rho^R} - \left(\frac{\partial^2 W^{RI}(x,t)}{\partial \rho^R\partial m_1^I}\cdot\Delta m_1 + \frac{\partial^2 W^{RI}(x,t)}{\partial \rho^R\partial m_2^I}\cdot\Delta m_2 + \frac{\partial^2 W^{RI}(x,t)}{\partial \rho^R\partial m_v^I}\cdot\Delta m_v\right)\right\}^2\cdot\sigma_\rho^2 \tag{10}$$

$$\overline{\sigma^2\left(W^{RI}(x,t)\right)} = \left\{\frac{\partial W^{RI}(x,t)}{\partial E^R} + \left(\frac{\partial^2 W^{RI}(x,t)}{\partial E^R\partial m_1^I}\cdot\Delta m_1 + \frac{\partial^2 W^{RI}(x,t)}{\partial E^R\partial m_2^I}\cdot\Delta m_2 + \frac{\partial^2 W^{RI}(x,t)}{\partial E^R\partial m_v^I}\cdot\Delta m_v\right)\right\}^2\cdot\sigma_E^2$$
$$+\left\{\frac{\partial W^{RI}(x,t)}{\partial I^R} + \left(\frac{\partial^2 W^{RI}(x,t)}{\partial I^R\partial m_1^I}\cdot\Delta m_1 + \frac{\partial^2 W^{RI}(x,t)}{\partial I^R\partial m_2^I}\cdot\Delta m_2 + \frac{\partial^2 W^{RI}(x,t)}{\partial I^R\partial m_v^I}\cdot\Delta m_v\right)\right\}^2\cdot\sigma_I^2$$
$$+\left\{\frac{\partial W^{RI}(x,t)}{\partial \rho^R} + \left(\frac{\partial^2 W^{RI}(x,t)}{\partial \rho^R\partial m_1^I}\cdot\Delta m_1 + \frac{\partial^2 W^{RI}(x,t)}{\partial \rho^R\partial m_2^I}\cdot\Delta m_2 + \frac{\partial^2 W^{RI}(x,t)}{\partial \rho^R\partial m_v^I}\cdot\Delta m_v\right)\right\}^2\cdot\sigma_\rho^2 \tag{11}$$

Numerical Simulations

In this paper, the vehicle-bridge interaction model is demonstrated as the Figure 1. The bridge's parameters are considered as Gaussian random variables. The parameters of vehicle are treated as interval variables. The nominal values (mean/midpoint values) of system parameters taken in the numerical simulation are listed in Table 1. The unit of the bridge displacement response is meter in this paper.

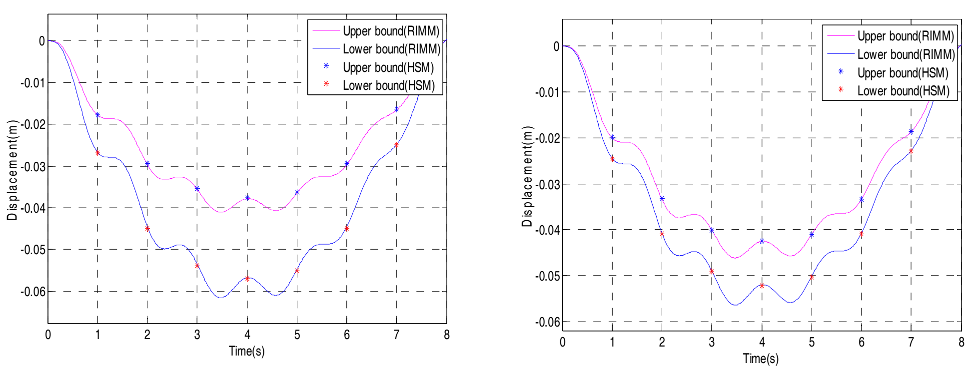
In this study, the bridge damping ratios ζ_{bj} for all modes are taken as 0.05. For the sake of simplicity, the coefficient of variation (COV) of ρ^R, E^R and I^R is adopted to represent the dispersal degree of random variables. Meanwhile, the interval change ratio (ICR) of m_1^I, m_2^I, and m_v^I is used to describe the scatter level of interval variables. vehicle speed, $v = 5m/s$ is taken into account to investigate the influence of vehicle velocity on the bridge response.

Table 1. Data of the vehicle-bridge model

Data of the bridge (mean value)	Data of the vehicle (midpoint)	
$L = 40m$	$m_1{}^c = 1000kg$	$m_2{}^c = 1500kg$
E=$33\ GN/m^2$	$m_v{}^c = 17800kg$	$I_v = 1.5\times10^5 kg.m^2$
$I = 0.16m^4$	$k_{s1} = 2.5\times10^6 N/m$	$k_{s2} = 4.2\times10^6 N/m$
$\rho = 7800kg/m$	$k_{t1} = 5.2\times10^6 N/m$	$k_{t2} = 7.2\times10^6 N/m$
	$a_1 = 0.52$	$a_2 = 0.48$
	$C_{s1} = 9000N/m$	$C_{s2} = 9600N/m$
	$C_{t1} = 920N/m$	$C_{t2} = 960N/m$
	$S = 4.27m$	

The mean value of the random interval bridge displacement response at its mid-span is given in Figures 2(a) (COV(ρ^R, E^R, I^R)=0.05, ICR(m_1^I, m_2^I, m_v^I)=0.2) and(b) (COV(ρ^R, E^R, I^R)=0.05, ICR(m_1^I, m_2^I, m_v^I)=0.1), when different combinations of uncertain parameters are taken. Figure 2 shows the mean bridge displacement response when the randomness of all random parameters and all interval parameters are considered. From Figure 2, it can be observed that the interval width of bridge response increases when the interval changes of interval variables become larger.

In summary, the mean value of the random interval bridge response is independent of the dispersal degrees of random system parameters as expected. The interval width of the mean value of bridge response is directly proportional to the uncertainties of interval variables and vehicle speed.



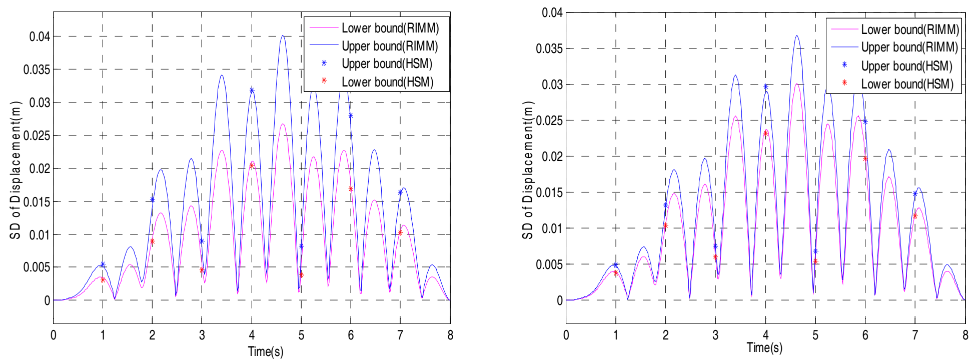
(a) (b)

Figure 2. Mean value of random interval bridge displacement response

The standard deviation (SD) of the random interval bridge displacement response at its mid-span is shown in Figures 3(a) and (b). It can also be observed that the interval width of the standard deviation of the random interval bridge response is directly proportional to the uncertainties of random and interval variables from Figures 3.

To validate the accuracy of the random interval moment method (RIMM) presented in this paper, a hybrid simulation method (HSM) is employed. This hybrid simulation method (HSM) combines direct simulation for interval variables and Monte-Carlo simulations for random variables.

To show the differences between the results generated by the RIMM and HSM in detail, the relative errors of mean value and standard deviation of bridge displacement are listed in Tables 2 and 3. Given the maximum relative error is 1.10% , while the coefficients of variation for all random parameters are 0.05 and the interval change ratios of all interval parameters are 0.2, the mean values calculated by the two methods are very closed to each other. For the standard deviation, the maximum relative error is 6.45%, which can be accepted because the hybrid simulation times used in this study are not enough to provide convergent results. 10,000 simulations used in the two rounds of HSM cannot yield convergent and reliable results although the total simulations are 10^6. The accuracy of the results obtained by the HSM can be improved if more simulations are implemented.



(a) (b)

Figure 3. Standard deviation of random interval bridge displacement

Generally, the accuracy of these results is satisfactory in practice. The presented random interval moment method has much less computational work than the simulation method. It should be noted that the accuracy of the results of random interval moment method can be further improved if second or higher order Taylor expansions are used.

Table 2. Comparison of mean values

Time (s)	Upper bound			Lower bound		
	RIMM	HSM	Error	RIMM	HSM	Error
1	0.02663	0.02671	0.29%	0.01793	0.01790	0.17%
2	0.04437	0.04461	0.55%	0.02958	0.02947	0.36%
3	0.05327	0.05353	0.49%	0.03550	0.03549	0.02%
4	0.05671	0.05680	0.16%	0.03781	0.03768	0.35%
5	0.05459	0.05467	0.14%	0.03639	0.03610	0.81%
6	0.04420	0.04436	0.37%	0.02961	0.02929	1.10%
7	0.02482	0.02506	0.97%	0.01636	0.01625	0.68%

Table 3. Comparison of standard deviations

Time	Upper bound			Lower bound		
	RIMM	HSM	Error	RIMM	HSM	Error
1	0.00521	0.00547	4.72%	0.00347	0.00339	2.33%
2	0.01394	0.01415	1.50%	0.00923	0.00894	3.22%
3	0.00886	0.00894	0.91%	0.00589	0.00577	2.14%
4	0.03162	0.03182	0.62%	0.02108	0.02095	0.63%
5	0.00852	0.00854	0.19%	0.00568	0.00549	3.54%
6	0.02617	0.02797	6.45%	0.01744	0.01713	1.83%
7	0.01596	0.01638	2.57%	0.01064	0.01024	3.87%

Conclusions

In this paper, stochastic dynamic response of vehicle-bridge interaction system with uncertainties is investigated by extending the random interval moment method to the dynamic coupling system. The uncertainties of system are modeled as random and interval variables. The expressions for calculating the bounds of expectation and variance of the random interval bridge response are derived. Using these formulations, the upper and lower bounds of mean value and standard deviation of bridge response can be very easily obtained. The results obtained by the presented random interval moment method are in very good agreement with those determined by Monte-Carlo simulation method. The relative errors of these two methods are quite small when the change ranges of system parameters are not large.

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