A time-domain BEM for dynamic crack analysis of magnetoelectroelastic composites

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Abstract

Transient dynamic crack analysis in two-dimensional, layered, anisotropic and linear magnetoelectroelastic solids is presented in this paper. A time-domain boundary element method (BEM) is developed for this purpose. The layered magnetoelectroelastic solids are modeled by the multi-domain formulation and the time-domain dynamic fundamental solutions for homogeneous linear magnetoelectroelastic solids are applied in the present BEM. A Galerkin-method is used for the spatial discretization of the boundary integral equations and a collocation method is implemented for the temporal discretization of the arising convolution integrals. An explicit time-stepping scheme is obtained to compute the discrete boundary data including the generalized crack-opening-displacements (CODs). Numerical examples are presented and discussed to show the effects of the interface, the material combinations and the dynamic loading on the intensity factors.

Keywords: time-domain BEM, magnetoelectroelastic composites, interior and interface cracks, dynamic intensity factors, impact loading.

Introduction

Due to their inherent coupling effects between mechanical, electrical and magnetic fields magnetoelectroelastic materials offer many possibilities for advanced smart structures (Nan, 1994). Layered or laminated composites are important applications of magnetoelectroelastic materials because they can be optimized to satisfy the high-performance requirements according to different in-service conditions. Interface cracks, may be induced by the mismatch of the mechanical, electric, magnetic and thermal properties of the material constituents during the manufacturing process and the in-service loading conditions, are one of the most dominant failure mechanisms in layered or laminated composites. Although the dynamic crack analysis in homogenous magnetoelectroelastic solids have been presented in several works (e.g., Sladek et al., 2008, Sladek et al., 2011, Wünsche et al., 2012) the analysis of interface cracks in layered magnetoelectroelastic solids is rather limited due to the problem complexity.

In this paper, the dynamic analysis of interface crack in two-dimensional, layered and linear magnetoelectroelastic solids under impact loading is presented. A time-domain boundary element method (TDBEM) is developed. The homogeneous magnetoelectroelastic layers are modeled by the multi-domain BEM formulation. The time-domain dynamic fundamental solutions for homogeneous and linear magnetoelectroelastic solids are applied in the present BEM. The spatial discretization of the boundary integral equations is performed by a Galerkin-method, while a collocation method is implemented for the temporal discretization of the arising convolution integrals. An explicit time-stepping scheme is applied to compute the discrete boundary data including the generalized crack-opening-displacements (CODs). In contrast to a crack inside a homogenous material the asymptotic crack-tip field for an interface crack between two dissimilar linear magnetoelectroelastic materials shows different kinds of oscillating and non-oscillating singularities. This makes an implementation of a special crack-tip element very difficult and therefore only standard elements are used at the crack-tips. A displacement-based extrapolation technique is applied to minimize the error in the computation of the dynamic intensity factors. To investigate the effects of the interface, the material combinations and the dynamic loading on the dynamic intensity factors, numerical examples are presented and discussed.

2 Problem formulation

Let us consider a piecewise homogeneous, layered and linear magnetoelectroelastic solid with an interface crack. In the absence of body forces, free electric charges, magnetic induction sources and applying the quasi-static assumption for the electric and magnetic fields, the cracked solid satisfies the generalized equations of motion

$$\sigma_{iJ,i}(\mathbf{x},t)=\rho^{\varsigma}\delta^{*}_{JK}\ddot{u}_{K}(\mathbf{x},t)\,,\ \delta^{*}_{JK}=\begin{cases}\delta_{JK},\, J,K=1,2\\ 0,\text{ otherwise}\end{cases} \tag{1}$$

and the constitutive equations

$$\sigma_{iJ}(\mathbf{x},t)=C^{\varsigma}_{iJKl}u_{K,l}(\mathbf{x},t)\,. \tag{2}$$

The generalized displacements u_I, the generalized stresses σ_{iJ} and the generalized elasticity matrix C_{iJKl} for a homogenous domains Ω^{ζ} (ζ=1,2,…,N) are defined by

$$u_I=\begin{cases}u_i,\, I=1,2\\ \varphi,\, I=4\\ \phi,\, I=5\end{cases}\,,\ \sigma_{iJ}=\begin{cases}\sigma_{iJ},\, J=1,2\\ D_i,\, J=4\\ B_i,\, J=5\end{cases}\,,\ C_{iJKl}=\begin{cases}c_{ijkl},\ J,K=1,2\\ e_{lij},\ J=1,2,\quad K=4\\ h_{lij},\ J=1,2,\quad K=5\\ e_{ikl},\ J=4,\quad K=1,2\\ -\varepsilon_{il},\ J,K=4\\ -\beta_{il},\ J=4,\quad K=5\\ h_{ikl},\ J=5,\quad K=1,2\\ -\beta_{il},\ J=5,\quad K=4\\ -\gamma_{il},\ J,K=5\end{cases}\,. \tag{3}$$

Further, the initial conditions

$$u_i(\mathbf{x},t=0)=\dot{u}_i(\mathbf{x},t=0)=0\,, \tag{4}$$

the boundary conditions

$$t_I(\mathbf{x},t)=\bar{t}_I(\mathbf{x},t)\,,\ \mathbf{x}\in\Gamma_t\,, \tag{5}$$

$$u_I(\mathbf{x},t)=\bar{u}_I(\mathbf{x},t)\,,\ \mathbf{x}\in\Gamma_u\,, \tag{6}$$

with t_I being the traction vector defined by

$$t_I(\mathbf{x},t)=\sigma_{jI}(\mathbf{x},t)e_j(\mathbf{x})\,, \tag{7}$$

and the continuity conditions on the interface except the crack-faces

$$u^{I}_{I}(\mathbf{x},t)=u^{II}_{I}(\mathbf{x},t)\,,\ \mathbf{x}\in\Gamma_{if}\,, \tag{8}$$

$$t^{I}_{I}(\mathbf{x},t)=-t^{II}_{I}(\mathbf{x},t)\,,\ \mathbf{x}\in\Gamma_{if} \tag{9}$$

are applied. In Eqs. (1)-(9), u_i, φ, Φ, σ_{ij}, D_i, B_i are the mechanical displacements, the electric potential, the magnetic potential, the mechanical stresses, the electric displacements and the magnetic inductions; ρ, c_{ijkl}, ε_{ij}, γ_{ij}, e_{ijk}, h_{ijk} and β_{ij} denote the mass density, the elasticity tensor, the dielectric permittivity tensor, the magnetic permittivity tensor, the piezoelectric tensor, the piezomagnetic tensor and the magnetoelectric tensor. Γ_t and Γ_u are the external boundaries where the generalized tractions t_I and the generalized displacements u_I are known and Γ_{if} is the interface between the homogenous domains Ω^ζ (ζ=1,2,…,N). In the present work, the interface cracks are considered as free of mechanical stresses, electric displacements and magnetic inductions with

$$\sigma_{iJ}(\mathbf{x} \in \Gamma_{c^+}, t) = \sigma_{iJ}(\mathbf{x} \in \Gamma_{c^-}, t) = 0 , \tag{10}$$

where $\Gamma_{c\pm}$ are the two crack-faces. The generalized crack-opening-displacements (CODs) are defined by

$$\Delta u_I(\mathbf{x}, t) = u_I(\mathbf{x} \in \Gamma_{c+}, t) - u_I(\mathbf{x} \in \Gamma_{c-}, t) \tag{11}$$

A comma after a quantity represents spatial derivatives while a dot over the quantity denotes time differentiation. Lower case Latin indices take the values 1 and 2 (elastic), while capital Latin indices take the values 1, 2 (elastic), 4 (electric) and 5 (magnetic).

3 Time-domain boundary integral equations

The initial-boundary value problem is solved with a spatial Galerkin-method. The time-domain BIEs for the generalized displacements and the generalized tractions can be written in a weighted residual sense as

$$\int_\Gamma \psi(\mathbf{x})\, u_J(\mathbf{x}, t)\, d\Gamma_x = \int_\Gamma \psi(\mathbf{x}) \int_\Gamma \left[u_{IJ}^G(\mathbf{x}, \mathbf{y}, t) * t_I(\mathbf{y}, t) - t_{IJ}^G(\mathbf{x}, \mathbf{y}, t) * u_I(\mathbf{y}, t) \right] d\Gamma_y d\Gamma_x \ , \tag{12}$$

$$\int_\Gamma \psi(\mathbf{x})\, t_J(\mathbf{x}, t)\, d\Gamma_x = \int_\Gamma \psi(\mathbf{x}) \int_\Gamma \left[v_{IJ}^G(\mathbf{x}, \mathbf{y}, t) * t_I(\mathbf{y}, t) - w_{IJ}^G(\mathbf{x}, \mathbf{y}, t) * u_I(\mathbf{y}, t) \right] d\Gamma_y d\Gamma_x \ , \tag{13}$$

where ψ is the weighting function, an asterisk " * " denotes the Riemann convolution, $u_{IJ}^G(\mathbf{x},\mathbf{y},t)$, $t_{IJ}^G(\mathbf{x},\mathbf{y},t)$, $v_{IJ}^G(\mathbf{x},\mathbf{y},t)$ and $w_{IJ}^G(\mathbf{x},\mathbf{y},t)$ are the generalized displacement, traction and higher-order traction fundamental solutions. The dynamic time-domain fundamental solutions for homogeneous, anisotropic and linear magnetoelectroelastic solids are not available in explicit form (Rojas-Díaz et al., 2008, Wünsche et al., 2012). Using the Radon transform technique the fundamental solutions in the 2D case can be defined by a line integral over a unit circle as

$$u_{IJ}^G(\mathbf{x}, \mathbf{y}, t) = \frac{H(t)}{4\pi^2} \int_{|\mathbf{n}|=1} \sum_{m=1}^{M} \frac{P_{IJ}^m}{\rho c_m} \frac{1}{c_m t + \mathbf{n} \cdot (\mathbf{y} - \mathbf{x})} d\mathbf{n} , \tag{14}$$

where H(t), $\mathbf{n}$, c_m and P_{IJ}^m are the Heaviside step function, the wave propagation vector, the phase velocities of the elastic waves and the projector (Wünsche et al., 2012). By integration by parts and applying the properties of the time convolution the time-domain generalized displacement fundamental solutions can be divided into a singular static and a regular dynamic part as

$$u_{IJ}^G(\mathbf{x}, \mathbf{y}, t) * f(t) = u_{IJ}^S(\mathbf{x}, \mathbf{y}) f(t) + u_{IJ}^D(\mathbf{x}, \mathbf{y}, t) * \dot{f}(t) . \tag{15}$$

Like the displacement fundamental solutions, the traction and the higher-order traction fundamental solutions can also be divided into their singular static and regular dynamic parts.

4 Dynamic intensity factors for an interface crack

The intensity factors for an interface crack between two dissimilar linear magnetoelectroelastic materials can be computed from the generalized crack-opening displacements (CODs)

$$\Delta \mathbf{u}(r, t) = (\mathbf{H} + \overline{\mathbf{H}}) \sqrt{\frac{r}{2\pi}} \left[\frac{K r^{i\varepsilon_1} \mathbf{w}}{(1 + 2i\varepsilon_1)\cosh(\pi\varepsilon_1)} + \frac{\overline{K} r^{-i\varepsilon_1} \overline{\mathbf{w}}}{(1 - 2i\varepsilon_1)\cosh(\pi\varepsilon_1)} + \frac{K_4 r^{-\varepsilon_2} \mathbf{w}_4}{(1 - 2\varepsilon_2)\cos(\pi\varepsilon_2)} + K_5 \mathbf{w}_5 \right], \tag{16}$$

where $K=K_1+iK_2$ is the complex stress intensity factor, K_4 is the electric displacement intensity factor and K_5 is the magnetic induction intensity factor, ε_1 and ε_2 are the bimaterial constants, an overbar denotes the complex conjugate and i stands for the imaginary unit. The complex Hermitian matrix $\mathbf{H}$ as well as the eigenvectors $\mathbf{w}$, $\mathbf{w}_4$ and $\mathbf{w}_5$ are defined by the material properties and can be computed similar to an interface crack between two piezoelectric materials (Suo et al., 1992).

5 Numerical solution algorithm

To solve the time-domain BIEs (12) and (13) numerically a solution procedure is presented in this section. The layered piecewise homogeneous and magnetoelectroelastic solids with interface cracks are dealt with by the multi-domain technique. A collocation method is used for the temporal discretization while the Galerkin-method is applied for the spatial discretization. For the spatial discretization, the crack-faces, the external boundary of each homogeneous sub-domain and the interfaces of the cracked magnetoelectroelastic solid are discretized by linear elements. All boundary integrations can be computed analytically by special techniques. Linear shape functions are used for the temporal discretization and all time integrations can also be performed analytically. Only the line integrals over the unit circle arising in the dynamic parts of the time-domain fundamental solutions need to be computed numerically by the standard Gaussian quadrature.
The asymptotic crack-tip field for an interface crack between two dissimilar magnetoelectroelastic materials shows different oscillating and non-oscillating singularities in the generalized stress field (Gao and Tong, 2003, Fan et al., 2009). This makes an implementation of special crack-tip elements rather difficult. For this reason, only standard elements are applied at the crack-tips for interface cracks. This is in contrast to crack-tips inside a homogeneous sub-domain. In this case special crack-tip elements can be implemented to describe the local square-root behavior of the generalized CODs near the crack-tips properly. This makes an accurate and a direct calculation of the intensity factors from the numerically computed CODs possible.
After temporal and spatial discretizations and considering the initial conditions the following systems of linear algebraic equations can be obtained for each sub-domain Ω^{ζ} (ζ=1,2,…,N)

$$\mathbf{C}_{\zeta}\,\mathbf{u}_{\zeta}^{K} = \mathbf{U}_{\zeta}^{S}\,\mathbf{t}_{\zeta}^{K} - \mathbf{T}_{\zeta}^{S}\,\mathbf{u}_{\zeta}^{K} + \mathbf{T}_{\zeta}^{S}\,\boldsymbol{\Delta}\mathbf{u}_{\zeta}^{K} + \sum_{k=1}^{K}\left[\mathbf{U}_{\zeta}^{D;K-k+1}\,\mathbf{t}_{\zeta}^{k} - \mathbf{T}_{\zeta}^{D;K-k+1}\,\mathbf{u}_{\zeta}^{k}\right], \tag{17}$$

$$\mathbf{D}_{\zeta}\,\mathbf{t}_{\zeta}^{K} = \mathbf{V}_{\zeta}^{S}\,\mathbf{t}_{\zeta}^{K} - \mathbf{W}_{\zeta}^{S}\,\mathbf{u}_{\zeta}^{K} + \mathbf{W}_{\zeta}^{S}\,\boldsymbol{\Delta}\mathbf{u}_{\zeta}^{K} + \sum_{k=1}^{K}\left[\mathbf{V}_{\zeta}^{D;K-k+1}\,\mathbf{t}_{\zeta}^{k} - \mathbf{W}_{\zeta}^{D;K-k+1}\,\mathbf{u}_{\zeta}^{k}\right]. \tag{18}$$

By invoking the continuity conditions (8) and (9) on the interface Γ_{if} and by considering the boundary conditions (5) and (6) as well as the crack-face boundary conditions (10), Eqs. (17) and (18) can be summarized and recast into a common system of linear algebraic equations

$$\mathbf{x}^{K} = (\mathbf{C}^{1})^{-1}\left[\mathbf{D}^{1}\,\mathbf{y}^{K} + \sum_{k=1}^{K-1}(\mathbf{B}^{K-k+1}\,\mathbf{t}^{k} - \mathbf{A}^{K-k+1}\,\mathbf{u}^{k})\right], \tag{19}$$

where $\mathbf{x}^K$ is the vector of the unknown boundary data, $\mathbf{y}^K$ represents the vector of the prescribed boundary data, $\mathbf{A}^k$, $\mathbf{B}^k$, $\mathbf{C}^1$ and $\mathbf{D}^1$ are the system matrices. Eq. (19) is an explicit time-stepping scheme and the unknown boundary data can be computed time-step by time-step.

6 Numerical examples

In the following, numerical examples are presented and discussed to show the effects of the interface, the mismatch of the material properties, the coupled fields and the dynamic loading on the intensity factors (IFs). To measure the intensity of the electric and magnetic loading the following loading parameters are introduced

$$\chi^{e} = \frac{e_{22}^{I}}{\varepsilon_{22}^{I}}\frac{D_0}{\sigma_0}, \quad \chi^{m} = \frac{h_{22}^{I}}{\gamma_{22}^{I}}\frac{D_0}{\sigma_0}, \tag{20}$$

where σ_0, D_0 and B_0 are the mechanical, electrical and magnetic loading amplitudes. For convenience of the presentation, the real part K_1 and the imaginary part K_2 of the complex dynamic stress intensity factors as well as the electric displacement intensity factor K_4 and the magnetic induction intensity factor K_5 of the interface crack are normalized by

$$K_1^*(t) = \frac{K_1(t)}{K_0}, \quad K_2^*(t) = \frac{K_2(t)}{K_0}, \quad K_4^*(t) = \frac{e_{22}^{I}}{\varepsilon_{22}^{I}}\frac{K_4(t)}{K_0}, \quad K_5^*(t) = \frac{h_{22}^{I}}{\gamma_{22}^{I}}\frac{K_5(t)}{K_0}, \tag{21}$$

where $K_0 = \sigma_0\sqrt{\pi a}$ with a being the half-length of an internal interface crack. In the same sense the dynamic intensity factors for a crack inside a homogenous layer, as defined in (Wünsche et al., 2012), are normalized by

$$K_I^*(t) = \frac{K_I(t)}{K_0}, \quad K_{II}^*(t) = \frac{K_{II}(t)}{K_0}, \quad K_{IV}^*(t) = \frac{e_{22}^{I}}{\varepsilon_{22}^{I}}\frac{K_{IV}(t)}{K_0}, \quad K_V^*(t) = \frac{h_{22}^{I}}{\gamma_{22}^{I}}\frac{K_V(t)}{K_0}. \tag{22}$$

As example a rectangular, layered and linear magnetoelectroelastic plate with a central interface crack of length 2a as shown in Figure 1 is considered. The poling directions are normal to the interface crack. The geometrical data are h=20.0mm, w=10.0mm and 2a=4.8mm.

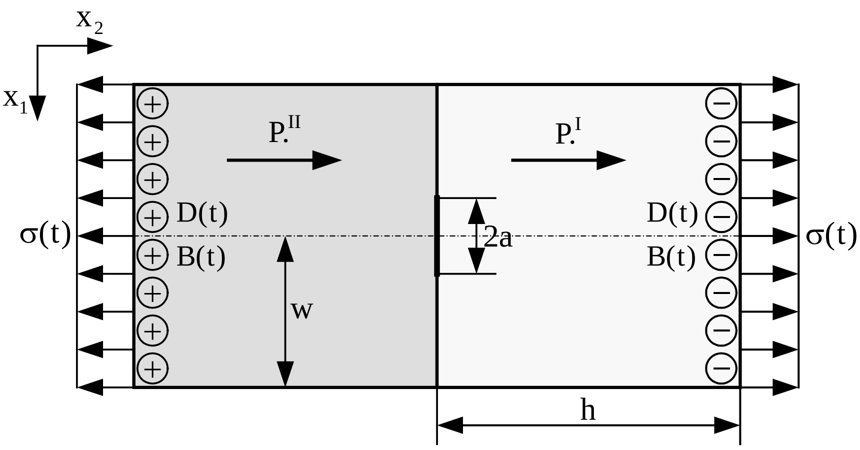

Figure 1. An interface crack in a rectangular layered magnetoelectroelastic plate

On the left and the right boundary of the cracked plate an impact tensile loading $\sigma(t)=\sigma_0 H(t)$, an impact electric loading $D(t)=D_0 H(t)$ and an impact magnetic loading $B(t)=B_0 H(t)$ is applied. The external boundary and the interface are discretized by an element-length of 1.0mm and each crack-face is approximated by 20 elements. A normalized time-step of $c_L\Delta t/h=0.05$ is chosen, where c_L is the quasi-longitudinal wave velocity. As material a $BaTiO_3$-$CoFe_2O_4$ composite, with $BaTiO_3$ being the piezoelectric phase and $CoFe_2O_4$ the piezomagnetic phase, is used (Nan, 1994). Figure 2 shows the numerical results of the present time-domain BEM obtained for different loadings and $BaTiO_3$-$CoFe_2O_4$ with a volume fraction $v_f=0.5$ for both layers. This special case is equal to an interior crack inside a homogenous magnetoelectroelastic plate. The normalized dynamic intensity factors for an interface crack between to magnetoelectroelastic layers with volume fractions of $v_f=0.5$ for domain I and $v_f=0.2$ for domain II are presented in Figure 3.

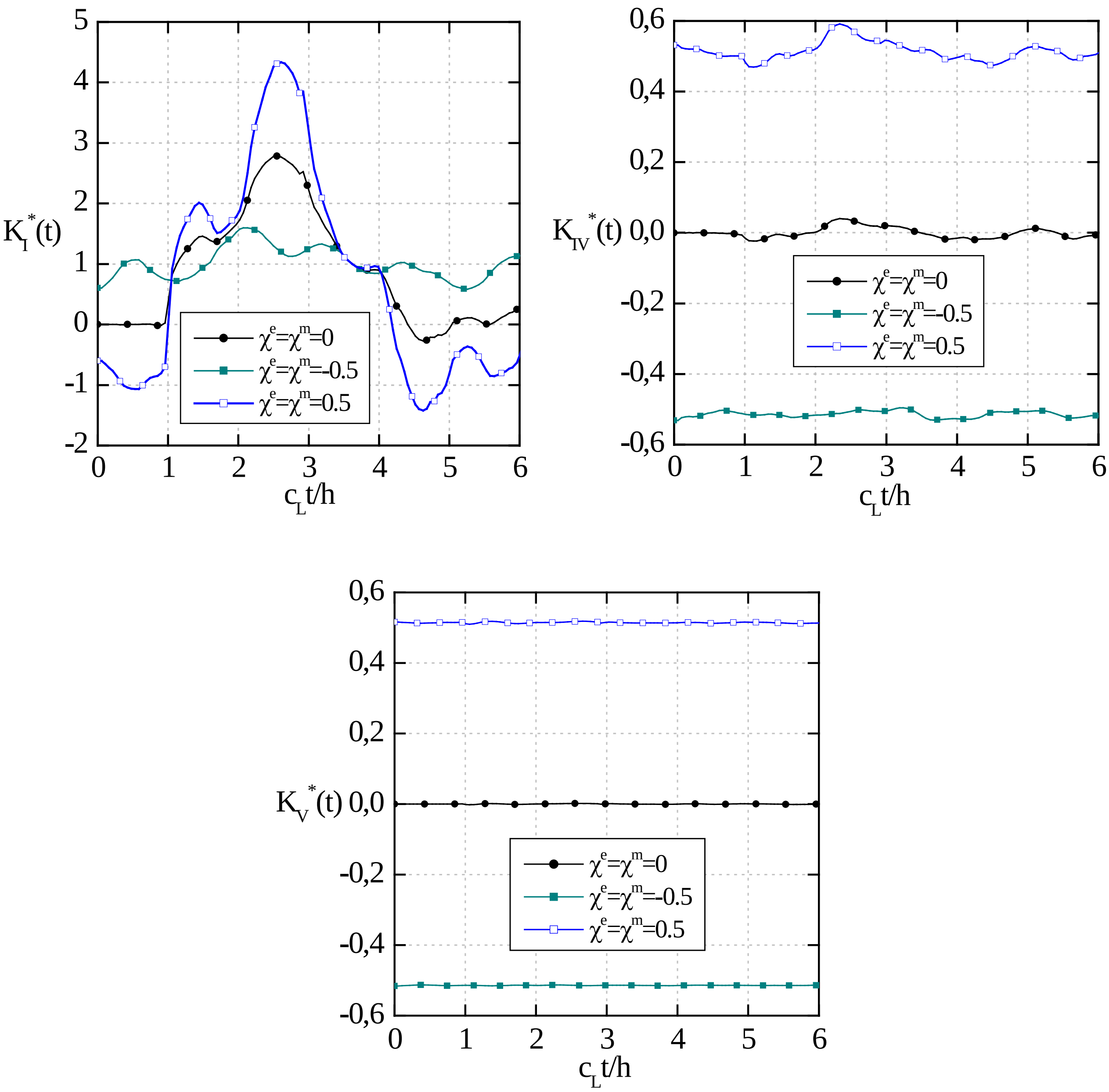

Figure 2. Normalized dynamic intensity factors for an interior crack subjected to different loadings

Figures 2 indicate that, if an electric and magnetic impact is applied, the normalized dynamic mode-I stress intensity factor starts from a non-zero value at t=0. This is due to the quasi-static assumption on the electromagnetic fields, which implies that the cracked magnetoelectroelatic plate is immediately subjected to an electromagnetic wave and as a consequence the crack opens at t=0. In contrast, the elastic waves induced by the mechanical impact need some time to reach the crack, as clearly observed for the case $\chi^e=\chi^m=0$. The peak values of the normalized dynamic intensity factors decrease with increasing electric and magnetic loading amplitudes. The dynamic mode-II intensity factors vanish, since no shear stress components are induced for all applied loadings normal to the crack-faces in the case of a transversely isotropic material behavior.

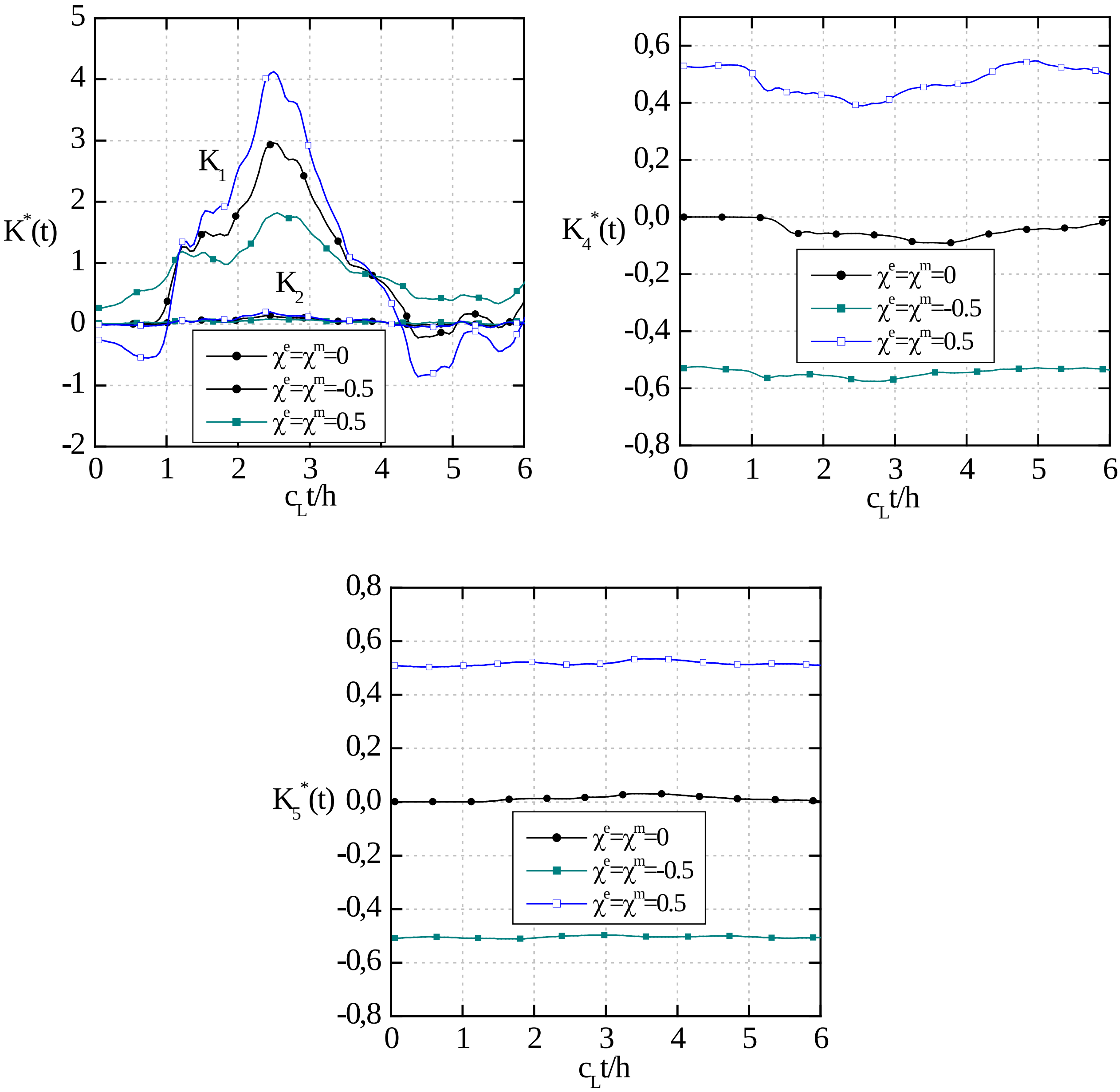

Figure 3. Normalized dynamic intensity factors for an interface crack subjected to different loadings

The real part of the complex intensity factor, the electrical displacement intensity factor and the magnetic induction intensity factor for the interface crack, as shown in Figure 3, have a similar global behavior than the dynamic mode-I, mode-IV and mode-V intensity factors for an interior crack in a homogenous magnetoelectroelastic plate. In contrast to the homogenous case, the crack opening and sliding modes I and II are coupled each other for the interface crack and therefore the imaginary part of the complex intensity factor is unequal zero. In can be observed, that the applied electric and magnetic loading may lead to a physically meaningless crack-face intersection in different time ranges for the case $\chi^e=\chi^m=0.5$. This requires an advanced iterative solution procedure for the crack-face contact analysis which is not considered in this work.

Conclusions

Transient dynamic crack analysis in layered and linear magnetoelectroelastic solids is presented in this work. For this purpose, a time-domain BEM is developed which uses a Galerkin-method for the spatial discretization and a collocation method for the temporal discretization. Both temporal and spatial integrations are carried out analytically. Only the line integrals over the unit circle in the dynamic fundamental solutions are computed numerically. An explicit time-stepping scheme is obtained for computing the unknown boundary data. Since the generalized displacement field for a crack in the interface between two dissimilar magnetoelectroelastic materials shows different oscillating and non-oscillating singularities the intensity factors are computed by a displacement extrapolation technique. The presented numerical examples indicate a significant influence of the interface, the material combination and the dynamic loading condition on the dynamic intensity factors.

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