Simultaneous Size and Shape Structural Optimization using Enhanced Comprehensive Learning Particle Swarm Optimization

***Soviphou Muong[1], Thu Huynh Van[1], Chung Nguyen Van[1, 2], and †Sawekchai Tangaramvong[1]**

[1] Applied Mechanics and Structures Research Unit, Department of Civil Engineering, Chulalongkorn University, Bangkok 10330, Thailand.

[2] Faculty of Civil Engineering, HCMC University of Technology and Education, Ho Chi Minh 721400, Vietnam.

*Presenting author: soviphou@gmail.com
†Corresponding author: sawekchai.t@chula.ac.th

Abstract

The paper proposes an enhanced version of the comprehensive learning particle swarm optimization (CLPSO) method for the simultaneous optimal size and shape design of steel truss structures under applied forces. The CLPSO approach incorporates the two novel enhancing techniques, namely perturbation-based exploitation and adaptive learning probabilities, in addition to its distinctive diversity of particles preventing the premature local optimum solutions. In essence, the perturbation enables the robust exploitation of the updating velocity of particles, whilst the learning probabilities are dynamically adjusted by the ranking information of personal best particles. A combination of these techniques results in the fast convergence and likelihood of the global optimum solution. Applications of the enhanced CLPSO method are illustrated through a number of successfully solved truss design examples. The robustness and accuracy of the proposed scheme are evidenced by the comparisons with available benchmarks processed by some other metaheuristic algorithms in obtaining the optimal size and shape distributions of steel trusses complying with limit state specifications.

Keywords: Comprehensive Learning; Particle Swarm Optimization; Perturbation-Based Exploitation; Adaptive Learning Probabilities; Size and Shape Optimization

Introduction

Particle swarm optimization (PSO) first introduced by Kennedy and Eberhart [1] in 1995 emulates the movement or social behavior of a bird flock. An PSO is a popular swarm-intelligence-based algorithm which is used in many real-world optimization problems. However, the solutions of complex problems can often be trapped in the local optima. Many approaches have been studied to improve the performance of PSO methods. In 2014, Xiang Yu and Xueqing Zhang [3] proposed an enhanced comprehensive learning PSO (ECLPSO) based on the original concept of comprehensive learning PSO (CLPSO) [2], where two enhancing techniques construct the perturbation-based exploitation together with adaptive learning probabilities.

This paper presents the novel ECLPSO method in the simultaneous size and shape optimization of planar truss structures. By considering both size and shape variables, the optimization provides the more economical material design than the size optimization alone. The applications of the proposed ECLPSO and its accuracy in obtaining the optimal solution are illustrated through the comparisons with some available benchmarks.

State Optimization Problem

The weight (cost) minimization problem of the truss structure consisting of *n* pin-connected members can be mathematically formulated in two design size (namely $\mathbf{X}_A \in \Re^n = \{A_1, \ldots, A_n\}$) and shape ($\mathbf{X}_G \in \Re^{ng} = \{G_1, \ldots, G_{ng}\}$) variables as follows:

$$\left.\begin{array}{lll}\text{find} & \mathbf{X} \in \Re^{n+ng} = \{\mathbf{X}_A, \mathbf{X}_G\} & \\ \text{minimize} & W\left(\mathbf{X}_A, \mathbf{X}_G\right) = \sum_{i=1}^{n} \rho_i L_i A_i & \\ \text{subject to} & \sigma_i^c \le \sigma_i \le \sigma_i^t & \text{for } \forall i \in \{1, \ldots, n\} \\ & \delta_{\min} \le \delta_j \le \delta_{\max} & \text{for } \forall j \in \{1, \ldots, m\} \\ & A_{\min} \le A_i \le A_{\max} & \text{for } \forall i \in \{1, \ldots, n\}\end{array}\right\}, \tag{1}$$

where W is the total weight of the design structure defined as the function of member density ρ_i, physical length L_i and cross-sectional area A_i, *m* the total number of degrees of freedom, *ng* the total number of geometry variables, δ_j the displacement at the *j*-th degree of freedom, and σ_i the member stress. The optimization problem in Eq. (1) minimizes the total weight W of the structure under the bounds on permissible compression σ_i^c and tension σ_i^t stresses, minimum $\delta_{\min}$ and maximum $\delta_{\max}$ displacements, and minimum $A_{\min}$ and maximum $A_{\max}$ areas.

The penalty method reformulates the constrained design Eq. (1) to an unconstrained optimization problem [7]-[9]. The function is defined below:

$$W' = W\left(\mathbf{X}_A, \mathbf{X}_G\right) \varphi_p K, \tag{2}$$

$$\varphi_p = \left(1 + C\right)^{\varepsilon}, \tag{3}$$

where K and ε are the penalty constant and penalty exponent (viz., $\varepsilon = 1$ in this study), C is the parameter measuring the violation of penalty constraints:

$$C = \sum_{j=1}^{m} C_\delta^j + \sum_{i=1}^{n} C_\sigma^i \tag{4}$$

C_δ^j and C_σ^i are the displacement and stress constraints, respectively:

$$\left.\begin{array}{ll} C_\delta^j = \left|\dfrac{\delta_j - \delta_{\min}}{\delta_{\min}}\right| & \text{if } \delta_j < \delta_{\min} \\ C_\delta^j = \left|\dfrac{\delta_j - \delta_{\max}}{\delta_{\max}}\right| & \text{if } \delta_j > \delta_{\max} \\ C_\delta^j = 0 & \text{if } \delta_{\min} < \delta_j < \delta_{\max} \end{array}\right\}, \tag{5}$$

$$\left.\begin{array}{ll} C_\sigma^i = \left|\dfrac{\sigma_i - \sigma_i^t}{\sigma_i^t}\right| & \text{if } \sigma_i > \sigma_i^t \\ C_\sigma^i = \left|\dfrac{\sigma_i - \sigma_i^c}{\sigma_i^c}\right| & \text{if } \sigma_i < \sigma_i^c \\ C_\sigma^i = 0 & \text{if } \sigma_i^c \le \sigma_i \le \sigma_i^t \end{array}\right\}. \tag{6}$$

Comprehensive Learning Particle Swarm Optimization

The CLPSO approach is a variant of the PSO algorithm pioneered by Liang and Qin [2] in 2006. The method employs the strategy that updates the particles' velocity by learning from all other particles' best information to prevent the solution from premature convergence. The velocity and position of a generic particle are determined by:

$$V_{i,d} = wV_{i,d} + cr_d(pbest_{f_i,d} - X_{i,d}) \tag{7}$$

$$X_{i,d} = X_{i,d} + V_{i,d} \tag{8}$$

where i and d are the array indices of the particles (population) and dimensions (design variables), respectively; $X_{i,d}$ and $V_{i,d}$ are the position and velocity of the *i*-th particle at the *d*-th dimension, respectively; w is the inertia weight; c is the acceleration coefficient being equal to 1.5; r_d is a random number in the range of $[0,1]$; f_i is the exemplar index the *i*-th particle follows; $pbest_{f_i,d}$ is the best position of the f_i-th particle for the *d*-th dimension.

$$L_i = L_{\min} + (L_{\max} - L_{\min})\frac{\exp\left(\dfrac{10(i-1)}{N-1}\right)-1}{\exp(10)-1}. \tag{9}$$

The selection of the exemplar index is based on the learning probability L_i. More explicitly, if the random number in a range of $[0,1]$ is greater than L_i then the exemplar index f_i reads the *i*-th particle's index. Otherwise, the index f_i takes the location of particle associated with the best of the two fitness values randomly selected from the populations of the considered *d*-th dimension.

The inertia weight is defined by:

$$w = w_{\max} - \frac{k}{k_{\max}}(w_{\max} - w_{\min}). \tag{10}$$

The maximum ($w_{\max}$) and minimum ($w_{\min}$) inertia weights are respectively equal to 0.9 and 0.4 for most cases, and $k_{\max}$ is the maximum generation number.

Enhanced Comprehensive Learning Particle Swarm Optimization (ECLPSO)

Two techniques, namely perturbation-based exploitation and adaptive learning probabilities, are developed within the ECLPSO such that the optimal sizes and shapes of structures are designed. These concepts are described as follows.

Perturbation-based Exploitation

The perturbation-based exploitation term mainly improves the exploitation and accuracy of CLPSO algorithm. The scheme is considered if Eq. (11) and Eq. (12) is true. Then, the perturbation term is added to the velocity equation in Eq. (7).

$$\overline{P}_d - \underline{P}_d \leq \alpha\left(\overline{X}_d - \underline{X}_d\right) \quad (11)$$

$$\overline{P}_d - \underline{P}_d \leq \beta \quad (12)$$

$$V_{i,d} = w_{PbE} V_{i,d} + cr_d\left(pbest_{f_i,d} + \eta\left(\frac{\overline{P}_d - \underline{P}_d}{2} - pbest_{f_i,d}\right) - X_{i,d}\right) \quad (13)$$

where $\overline{P}_d$ and $\underline{P}_d$ are the upper and lower bounds of the personal best position on the *d*-th dimension, respectively; $\overline{X}_d$ and $\underline{X}_d$ are respectively the upper and lower bounds of the search space; α is a relative ratio equal to 0.01; β is the small absolute bound with value equals to 2; w_{PbE} is the inertia weight for the approach exploitation (w_{PbE} = 0.5 in this work); η is the perturbation coefficient assisting the supports optimization process to capture the global optimum.

Adaptive Learning Probabilities

A new adaptive learning probabilities strategy is introduced to replace the original function in Eq. (9) as follows:

$$L_i = L_{\min} + (L_{\max} - L_{\min})\frac{\exp\left(\frac{10(K_i - 1)}{N-1}\right) - 1}{\exp(10) - 1}, \quad (14)$$

$$L_{\max} = L_{\min} + 0.25 + 0.45\log_{(D+1)}(M_k - 1). \quad (15)$$

The value K_i is a rank of personal best fitness value for the *i*-th particle, defined in an ascending order of the personal best fitness values. Moreover, $L_{\min}$ is fixed at 0.05, and M_k is the number of dimensions as when both Eqs. (11) and (12) are complied.

Illustrative Example

The example considers the 15-bar planar truss structure in Fig. 1 subjected to the vertical load of $F = 10$ kips. The material properties employed were the modulus of elasticity of 10^4 ksi, the material density of 0.1 lb.in^{-3}, and the permissible tensile and compressive stresses of $\sigma_i^t = \sigma_c^t = 25$ ksi for all members $i \in \{1, \ldots, 15\}$.

The sizing and shape optimization problem in Eq. (1) defined the design variables, namely the unknown member areas of $W_A = [A_1, \ldots, A_{15}]$ and some coordinates variations in *x*- and *y*-axes of $W_G = [x_2, x_3, x_6, x_7, y_2, y_3, y_4, y_6, y_7, y_8]$, where the geometry constraints of $x_2 = x_6$ and $x_3 = x_7$ were imposed. Moreover, the limits on both geometry variables are listed in Table 1.

The area variables are selected from the set available sections consisting of the discrete areas of A = {0.111, 0.141, 0.174, 0.220, 0.270, 0.287, 0.347, 0.440, 0.539, 0.954, 1.081, 1.174, 1.333, 1.488, 1.764, 2.142, 2.697, 2.800, 3.131, 3.565, 3.813, 4.805, 5.952, 6.572, 7.192, 8.525, 9.300, 10.850, 13.330, 14.290, 17.170, 19.180} in^2

Table 1. Limit on geometry variables

Geometry variables:	X_G, min (in)	X_G, max (in)
x_2	100	140
x_3	220	260
y_2	100	140
y_3	100	140
y_4	50	90
y_6	-20	20
y_7	-20	20
y_8	20	60

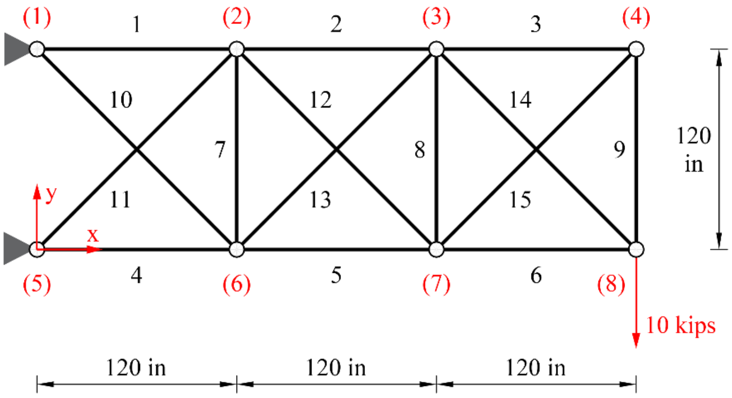

Figure 1. Schematic of the planar 15-bar truss structure.

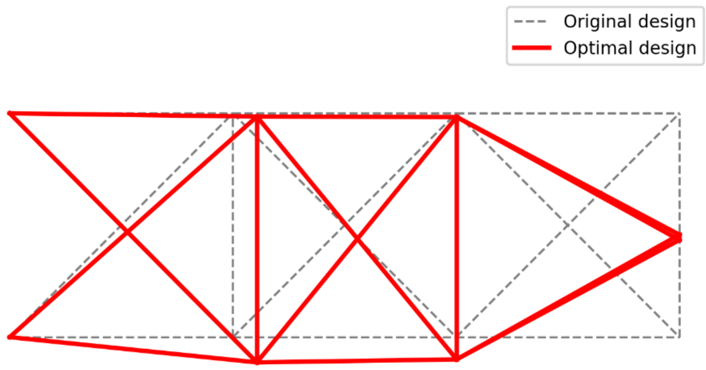

Figure 2. Optimum layout of the 15-bar truss structure.

The ECLPSO method adopted 30 particles with the maximum number of 800 iterations. All imposed constraints were fully complied. The resulting optimal member areas and shapes of the planar truss are depicted in Table 2, where those from various design methods are also summarized. It is evidenced that the optimal design weight value of 75.1552 lb given by the present ECLPSO achieved the most minimum (viz., the economical design with some 8% lighter than that from standard PSO scheme) as compared to all other benchmarks. The optimal design shape is depicted in Fig. 2, and the plot of solution convergence in Fig. 3 presents monotonic variations of the design weights decreasing to the optimum over the increasing number of iterations.

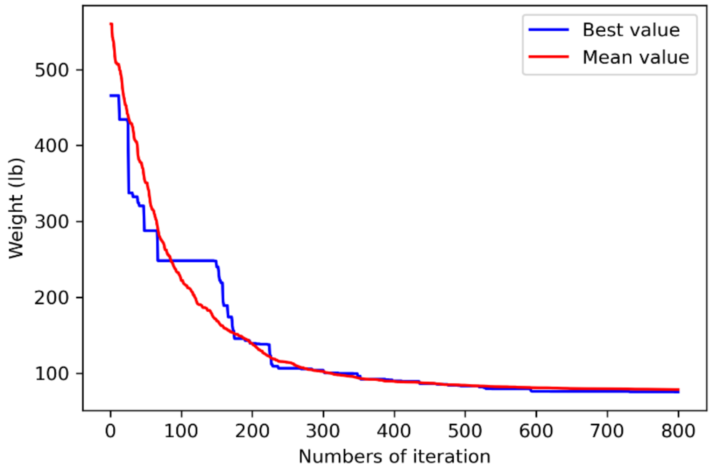

Figure 3. Solution convergence in the ECLPSO process.

Table 2. Optimal area and shape solutions computed various design methods.

Design variable	Rahami et al. [4]	Tang et al. [5]	Gholizadh [6]		ECLPSO (this study)	
			PSO	CPSO		Stress (ksi)
A1	1.081	1.081	0.954	1.174	0.954	24.9120
A2	0.539	0.539	1.081	0.539	0.539	23.8234
A3	0.287	0.287	0.270	0.347	0.174	24.8164
A4	0.954	0.954	1.081	0.954	0.954	-23.4293
A5	0.539	0.954	0.539	0.954	0.539	-24.8863
A6	0.141	0.220	0.287	0.141	0.287	-24.2336
A7	0.111	0.111	0.141	0.141	0.111	-2.0217
A8	0.111	0.111	0.111	0.111	0.141	-4.3943
A9	0.539	0.287	0.347	1.174	0.539	7.0051
A10	0.440	0.220	0.440	0.141	0.347	24.9239
A11	0.539	0.440	0.270	0.440	0.440	-24.2596
A12	0.270	0.440	0.111	0.440	0.270	20.0072
A13	0.220	0.111	0.347	0.141	0.174	-24.9816
A14	0.141	0.220	0.440	0.141	0.287	24.9373
A15	0.287	0.347	0.220	0.347	0.174	-24.3701
X2	101.5775	133.612	106.0521	102.2873	132.8913	
X3	227.9112	234.752	239.0245	240.5050	240.2414	
Y2	134.7986	100.449	130.3556	112.5840	118.2489	
Y3	128.2206	104.738	114.2730	108.0428	118.0375	
Y4	54.8630	73.762	51.9866	57.7952	52.0899	
Y6	-16.4484	-10.067	1.8135	6.4299	-13.5400	
Y7	-16.4484	-1.339	9.1827	1.8006	-11.9345	
Y8	54.8572	50.402	46.9087	57.7987	54.9136	
Weight (lb)	76.6854	79.820	82.2344	77.6153	75.1552	

Concluding Remarks

The paper has presented the simultaneous sizing and shape optimization of the in-plane truss structures under limited stress and serviceability constraints. The ECLPSO shows its efficient and robust optimizer that incorporates the two enhancing techniques, called the perturbation-based exploitation and the adaptive learning probabilities. The scheme by adjusting the ranking of personal best information advantageously overcomes the burdens associated with the premature solution convergence as would be experienced in standard PSO methods. The applications of the method have been illustrated through the design of modest-scale sizing and shape truss designs given in some available benchmarks. The optimal design solution can be achieved with the fast convergence to the optimum by processing the ECLPSO approach. The accuracy of the results computed has been well compared with those reported in the literatures.

Acknowledgements

This research has been supported under the Thailand Research Fund (Grant No. RSA6280089). The financial support provided by Chulalongkorn University under Ratchadaphiseksomphot Endowment Fund is acknowledged.

References

[1] Kennedy, J. and R. Eberhart (1995). Particle swarm optimization. Proceedings of ICNN'95 - International Conference on Neural Networks.

[2] Liang, J. J., A. K. Qin, P. N. Suganthan and S. Baskar (2006). "Comprehensive learning particle swarm optimizer for global optimization of multimodal functions." IEEE Transactions on Evolutionary Computation 10(3): 281-295.

[3] Yu, X. and X. Zhang (2014). "Enhanced comprehensive learning particle swarm optimization." Applied Mathematics and Computation 242: 265-276.

[4] Rahami, H., A. Kaveh and Y. Gholipour (2008). "Sizing, geometry and topology optimization of trusses via force method and genetic algorithm." Engineering Structures 30(9): 2360-2369.

[5] Tang, W., L. Tong and Y. Gu (2005). "Improved genetic algorithm for design optimization of truss structures with sizing, shape and topology variables." International Journal for Numerical Methods in Engineering 62(13): 1737-1762.

[6] Gholizadeh, S. (2013). "Layout optimization of truss structures by hybridizing cellular automata and particle swarm optimization." Computers & Structures 125: 86-99.

[7] Rajeev, S. and C. S. Krishnamoorthy (1992). "Discrete Optimization of Structures Using Genetic Algorithms." Journal of Structural Engineering 118(5): 1233-1250.

[8] Farshchin, M., C. V. Camp and M. Maniat (2016). "Optimal design of truss structures for size and shape with frequency constraints using a collaborative optimization strategy." Expert Systems with Applications 66: 203-218.

[9] Jawad, F. K. J., C. Ozturk, W. Dansheng, M. Mahmood, O. Al-Azzawi and A. Al-Jemely (2021). "Sizing and layout optimization of truss structures with artificial bee colony algorithm." Structures 30: 546-559.