Simultaneous Size and Shape Structural Optimization using Enhanced Comprehensive Learning Particle Swarm Optimization

*Soviphou Muong¹, Thu Huynh Van¹, Chung Nguyen Van¹,², and †Sawekchaisong Tangaramvong¹

¹Applied Mechanics and Structures Research Unit, Department of Civil Engineering, Chulalongkorn University, Bangkok 10330, Thailand.
²Faculty of Civil Engineering, HCMC University of Technology and Education, Ho Chi Minh 721400, Vietnam.

*Presenting author: soviphou@gmail.com
†Corresponding author: sawekchais@chula.ac.th

Abstract

The paper proposes an enhanced version of the comprehensive learning particle swarm optimization (CLPSO) method for the simultaneous optimal size and shape design of steel truss structures under applied forces. The CLPSO approach incorporates the two novel enhancing techniques, namely perturbation-based exploitation and adaptive learning probabilities, in addition to its distinctive diversity of particles preventing the premature local optimum solutions. In essence, the perturbation enables the robust exploitation of the updating velocity of particles, whilst the learning probabilities are dynamically adjusted by the ranking information of personal best particles. A combination of these techniques results in the fast convergence and likelihood of the global optimum solution. Applications of the enhanced CLPSO method are illustrated through a number of successfully solved truss design examples. The robustness and accuracy of the proposed scheme are evidenced by the comparisons with available benchmarks processed by some other metaheuristic algorithms in obtaining the optimal size and shape distributions of steel trusses complying with limit state specifications.

Keywords: Comprehensive Learning; Particle Swarm Optimization; Perturbation-Based Exploitation; Adaptive Learning Probabilities; Size and Shape Optimization

Introduction

Particle swarm optimization (PSO) first introduced by Kennedy and Eberhart [1] in 1995 emulates the movement or social behavior of a bird flock. An PSO is a popular swarm-intelligence-based algorithm which is used in many real-world optimization problems. However, the solutions of complex problems can often be trapped in the local optima. Many approaches have been studied to improve the performance of PSO methods. In 2014, Xiang Yu and Xueqing Zhang [3] proposed an enhanced comprehensive learning PSO (ECLPSO) based on the original concept of comprehensive learning PSO (CLPSO) [2], where two enhancing techniques construct the perturbation-based exploitation together with adaptive learning probabilities.

This paper presents the novel ECLPSO method in the simultaneous size and shape optimization of planar truss structures. By considering both size and shape variables, the optimization provides the more economical material design than the size optimization alone. The applications of the proposed ECLPSO and its accuracy in obtaining the optimal solution are illustrated through the comparisons with some available benchmarks.
State Optimization Problem

The weight (cost) minimization problem of the truss structure consisting of \( n \) pin-connected members can be mathematically formulated in two design size (namely \( X_A \in \mathbb{R}^n = \{A_1, \ldots, A_n\} \)) and shape (\( X_G \in \mathbb{R}^{ng} = \{G_1, \ldots, G_{ng}\} \)) variables as follows:

\[
\begin{align*}
\text{find} & \quad X \in \mathbb{R}^{n+ng} = \{X_A, X_G\} \\
\text{minimize} & \quad W(X_A, X_G) = \sum_{i=1}^{n} \rho_i L_i A_i \\
\text{subject to} & \quad \sigma^c \leq \sigma_i \leq \sigma^t \quad \text{for} \quad \forall i \in \{1, \ldots, n\} \\
& \quad \delta^\text{min} \leq \delta_j \leq \delta^\text{max} \quad \text{for} \quad \forall j \in \{1, \ldots, m\} \\
& \quad A^\text{min} \leq A_i \leq A^\text{max} \quad \text{for} \quad \forall i \in \{1, \ldots, n\}
\end{align*}
\]

where \( W \) is the total weight of the design structure defined as the function of member density \( \rho_i \), physical length \( L_i \) and cross-sectional area \( A_i \), \( m \) the total number of degrees of freedom, \( ng \) the total number of geometry variables, \( \delta_j \) the displacement at the \( j \)-th degree of freedom, and \( \sigma_i \) the member stress. The optimization problem in Eq. (1) minimizes the total weight \( W \) of the structure under the bounds on permissible compression \( \sigma^c \) and tension \( \sigma^t \) stresses, minimum \( \delta^\text{min} \) and maximum \( \delta^\text{max} \) displacements, and minimum \( A^\text{min} \) and maximum \( A^\text{max} \) areas.

The penalty method reformulates the constrained design Eq. (1) to an unconstrained optimization problem [7]-[9]. The function is defined below:

\[
W' = W(X_A, X_G) \varphi \rho K,
\]

\[
\varphi = (1 + C)^\varepsilon,
\]

where \( K \) and \( \varepsilon \) are the penalty constant and penalty exponent (viz., \( \varepsilon = 1 \) in this study), \( C \) is the parameter measuring the violation of penalty constraints:

\[
C = \sum_{j=1}^{m} C^j + \sum_{i=1}^{n} C^i
\]

\( C^j \) and \( C^i \) are the displacement and stress constraints, respectively:

\[
C^j = \begin{cases} 
\left| \frac{\delta_j - \delta^\text{min}}{\delta^\text{min}} \right| & \text{if} \quad \delta_j < \delta^\text{min} \\
\left| \frac{\delta_j - \delta^\text{max}}{\delta^\text{max}} \right| & \text{if} \quad \delta_j > \delta^\text{max} \\
0 & \text{if} \quad \delta^\text{min} < \delta_j < \delta^\text{max}
\end{cases}
\]

(5)
\[
C'_{\alpha} = \begin{cases} 
\frac{\sigma_i - \sigma'_i}{\sigma'_i} & \text{if } \sigma_i > \sigma'_i \\
\frac{\sigma_i - \sigma^c_i}{\sigma^c_i} & \text{if } \sigma_i < \sigma^c_i \\
0 & \text{if } \sigma^c_i \leq \sigma_i \leq \sigma'_i 
\end{cases}.
\]

Comprehensive Learning Particle Swarm Optimization

The CLPSO approach is a variant of the PSO algorithm pioneered by Liang and Qin [2] in 2006. The method employs the strategy that updates the particles’ velocity by learning from all other particles’ best information to prevent the solution from premature convergence. The velocity and position of a generic particle are determined by:

\[
V_{i,d} = wV_{i,d} + c r_d (pbest_{f_i,d} - X_{i,d})
\]

\[
X_{i,d} = X_{i,d} + V_{i,d}
\]

where \(i\) and \(d\) are the array indices of the particles (population) and dimensions (design variables), respectively; \(X_{i,d}\) and \(V_{i,d}\) are the position and velocity of the \(i\)-th particle at the \(d\)-th dimension, respectively; \(w\) is the inertial weight; \(c\) is the acceleration coefficient being equal to 1.5; \(r_d\) is a random number in the range of \([0,1]\); \(f_i\) is the exemplar index the \(i\)-th particle follows; \(pbest_{f_i,d}\) is the best position of the \(f_i\)-th particle for the \(d\)-th dimension.

\[
L_i = L_{\min} + (L_{\max} - L_{\min}) \frac{\exp\left(\frac{10(i-1)}{N-1}\right) - 1}{\exp(10) - 1}.
\]

The selection of the exemplar index is based on the learning probability \(L_i\). More explicitly, if the random number in a range of \([0,1]\) is greater than \(L_i\) then the exemplar index \(f_i\) reads the \(i\)-th particle’s index. Otherwise, the index \(f_i\) takes the location of particle associated with the best of the two fitness values randomly selected from the populations of the considered \(d\)-th dimension.

The inertia weight is defined by:

\[
w = w_{\max} - \frac{k}{k_{\max}} (w_{\max} - w_{\min}).
\]

The maximum \((w_{\max})\) and minimum \((w_{\min})\) inertia weights are respectively equal to 0.9 and 0.4 for most cases, and \(k_{\max}\) is the maximum generation number.

Enhanced Comprehensive Learning Particle Swarm Optimization (ECLPSO)

Two techniques, namely perturbation-based exploitation and adaptive learning probabilities, are developed within the ECLPSO such that the optimal sizes and shapes of structures are designed. These concepts are described as follows.
Perturbation-based Exploitation

The perturbation-based exploitation term mainly improves the exploitation and accuracy of CLPSO algorithm. The scheme is considered if Eq. (11) and Eq. (12) is true. Then, the perturbation term is added to the velocity equation in Eq. (7).

\[ \overline{P}_d - \underline{P}_d \leq \alpha (\overline{X}_d - \underline{X}_d) \]  \hspace{1cm} (11)

\[ \overline{P}_d - \underline{P}_d \leq \beta \]  \hspace{1cm} (12)

\[ V_{i,d} = w_{pBE} V_{i,d} + c_r d \left( p_{best_i,d} + \eta \left( \frac{\overline{P}_d - \underline{P}_d}{2} - p_{best_i,d} \right) - X_{i,d} \right) \]  \hspace{1cm} (13)

where \( \overline{P}_d \) and \( \underline{P}_d \) are the upper and lower bounds of the personal best position on the \( d \)-th dimension, respectively; \( \overline{X}_d \) and \( \underline{X}_d \) are respectively the upper and lower bounds of the search space; \( \alpha \) is a relative ratio equal to 0.01; \( \beta \) is the small absolute bound with value equals to 2; \( w_{pBE} \) is the inertia weight for the approach exploitation (\( w_{pBE} = 0.5 \) in this work); \( \eta \) is the perturbation coefficient assisting the supports optimization process to capture the global optimum.

Adaptive Learning Probabilities

A new adaptive learning probabilities strategy is introduced to replace the original function in Eq. (9) as follows:

\[ L_i = L_{\text{min}} + (L_{\text{max}} - L_{\text{min}}) \frac{\exp \left( \frac{10(K_i - 1)}{N - 1} \right) - 1}{\exp(10) - 1} \],  \hspace{1cm} (14)

\[ L_{\text{max}} = L_{\text{min}} + 0.25 + 0.45 \log_{e(P+1)}(M_k - 1). \]  \hspace{1cm} (15)

The value \( K_i \) is a rank of personal best fitness value for the \( i \)-th particle, defined in an ascending order of the personal best fitness values. Moreover, \( L_{\text{min}} \) is fixed at 0.05, and \( M_k \) is the number of dimensions as when both Eqs. (11) and (12) are complied.

Illustrative Example

The example considers the 15-bar planar truss structure in Fig. 1 subjected to the vertical load of \( F = 10 \) kips. The material properties employed were the modulus of elasticity of \( 10^4 \) ksi, the material density of 0.1 lb.in\(^{-3}\), and the permissible tensile and compressive stresses of \( \sigma'_{\text{t}} = \sigma'_{\text{c}} = 25 \) ksi for all members \( i \in \{1, \ldots, 15\} \).

The sizing and shape optimization problem in Eq. (1) defined the design variables, namely the unknown member areas of \( W_A = \{A_1, \ldots, A_{15}\} \) and some coordinates variations in \( x \)- and \( y \)-axes of \( W_G = \{x_2, x_3, x_6, x_7, y_2, y_3, y_4, y_6, y_7, y_9\} \), where the geometry constraints of \( x_2 = x_6 \) and \( x_3 = x_7 \) were imposed. Moreover, the limits on both geometry variables are listed in Table 1.
The area variables are selected from the set of available sections consisting of the discrete areas of \( A = \{0.111, 0.141, 0.174, 0.220, 0.287, 0.347, 0.440, 0.539, 0.954, 1.081, 1.174, 1.333, 1.488, 1.764, 2.142, 2.697, 2.800, 3.131, 3.565, 3.813, 4.805, 5.952, 6.572, 7.192, 8.525, 9.300, 10.850, 13.330, 14.290, 17.170, 19.180\} \text{in}^2.

<table>
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<tr>
<th>Geometry variables: ( x, y_{\text{min}} ) (in)</th>
<th>( x, y_{\text{max}} ) (in)</th>
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<tr>
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</tr>
<tr>
<td>( y_2 )</td>
<td>100</td>
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<tr>
<td>( y_3 )</td>
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<tr>
<td>( y_7 )</td>
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</tr>
<tr>
<td>( y_8 )</td>
<td>20</td>
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</tbody>
</table>

**Figure 1. Schematic of the planar 15-bar truss structure.**

**Figure 2. Optimum layout of the 15-bar truss structure.**

The ECLPSO method adopted 30 particles with the maximum number of 800 iterations. All imposed constraints were fully complied. The resulting optimal member areas and shapes of the planar truss are depicted in Table 2, where those from various design methods are also summarized. It is evidenced that the optimal design weight value of 75.1552 lb given by the present ECLPSO achieved the most minimum (viz., the economical design with some 8% lighter than that from standard PSO scheme) as compared to all other benchmarks. The optimal
design shape is depicted in Fig. 2, and the plot of solution convergence in Fig. 3 presents monotonic variations of the design weights decreasing to the optimum over the increasing number of iterations.

![Plot of solution convergence in the ECLPSO process.](image)

**Figure 3. Solution convergence in the ECLPSO process.**

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>A1</td>
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<td>1.081</td>
<td>0.954</td>
<td>1.174</td>
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<tr>
<td>A2</td>
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<td>0.287</td>
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<tr>
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<tr>
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<tr>
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<tr>
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<td>46.9087</td>
<td>57.7987</td>
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</tbody>
</table>
| Weight (lb)     | 76.6854           | 79.820          | 82.2344       | 77.6153             | 75.1552

*Table 2. Optimal area and shape solutions computed various design methods.*
Concluding Remarks

The paper has presented the simultaneous sizing and shape optimization of the in-plane truss structures under limited stress and serviceability constraints. The ECLPSO shows its efficient and robust optimizer that incorporates the two enhancing techniques, called the perturbation-based exploitation and the adaptive learning probabilities. The scheme by adjusting the ranking of personal best information advantageously overcomes the burdens associated with the premature solution convergence as would be experienced in standard PSO methods. The applications of the method have been illustrated through the design of modest-scale sizing and shape truss designs given in some available benchmarks. The optimal design solution can be achieved with the fast convergence to the optimum by processing the ECLPSO approach. The accuracy of the results computed has been well compared with those reported in the literatures.

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