A Turbulent Flow of Water-Based Optimization for the Optimal Sizing Design of Steel Trusses

***Saw Thiri Khaing[1], Thu Huynh Van[1], and †Sawekchai Tangaramvong[1]**

1 Applied Mechanics and Structures Research Unit, Department of Civil Engineering, Chulalongkorn University, 10330 Bangkok, Thailand

*Presenting author: sawthirikhaing@gmail.com
†Corresponding author: sawekchai.t@chula.ac.th

Abstract

The paper presents the novel meta-heuristic, called turbulent flow of water-based optimization, algorithm to determine the optimal distribution of steel member sizes allocated to the truss structure that can safely sustain the specified design forces. The problem states the minimization of the cost function, described by the total weight of the designed structure, complying with the limit state specifications. The TFWO method performs the random searches among various whirlpool sets, where the best particle position of each group is pulled down by the centripetal force to the cavity in the center of a whirlpool. A centrifugal force acting in an opposite direction to the centripetal force randomly transfers the particle to the new position. The interaction between different whirlpools applies the individual centripetal forces to iteratively unify those surrounding whirlpools into ones with stronger tractions, and subsequently converges the optimal design solution. The accuracy of the TFWO scheme is illustrated through comparisons with some benchmarks processed by various recent optimization algorithms. These examples present the robustness of the proposed approach in the optimal design of steel structures at modest computing resources.

Keywords: Turbulent Flow of Water-based Optimization, Structural Optimization, Meta-Heuristic Algorithm, Steel Structures, Optimal Sizing Design.

Introduction

The structural optimization determines the optimal distribution of members and sizes assigned to the structure under the required strength and serviceability performance criteria. The problem is typically written in the mathematical formulations aiming to computing the objective function (typically cost minimization) subjected to the constraints intrinsically describing the targeted design responses. The fast growing of recent computing technologies has encouraged the development of meta-heuristic methods that systematically perform the iterative-type design procedures to converge the optimal solutions. The methods are generally inspired by the concepts observing the nature-like collective birds and animal behaviors, e.g., genetic algorithm [1], particle swarm optimization (PSO) [2], artificial bee colony (ABC) [3], flower and big bang-big crunch [4] etc. On the other hand, one of the major drawbacks underlying is the return of local optimum leading to the premature solution convergence. The ability in obtaining the accurate optimal designs is largely problem dependent. The exploration of new and suitable methods is thus necessary for the specific structural design problems considered.

This study proposes the development of a so-called turbulent flow of water-based optimization (TFWO) to process the sizing design of steel trusses under the required forces[5]. The TFWO is inspired by the random behaviors in nature established for examples in rivers, seas and oceans. It provides the optimal solutions of various complex problems with real-parameter benchmark functions for different dimensions. The specific problem considers the cost (total weight) minimization as the objective function subject to the constraints on the limited strength and serviceability responses of the design structures.

Optimal Sizing Design Formulations

The minimum weight design of the pin-connected steel truss structure can be mathematically described as follows:

$$\text{Minimize} \qquad \mathrm{W} = \sum_{\mathrm{i}=1}^{\mathrm{n}} \rho_{\mathrm{i}} \mathrm{A}_{\mathrm{i}} \mathrm{L}_{\mathrm{i}} \tag{1}$$

$$\begin{aligned} \text{subject to} \qquad & \sigma_{\mathrm{i}} \leq \sigma_{\mathrm{all}},\ \mathrm{i} = 1,2, \text{------},\mathrm{n} \\ & \delta_{\mathrm{i}} \leq \delta_{\mathrm{all}},\ \mathrm{i} = 1,2, \text{------},\mathrm{n} \end{aligned} \tag{2}$$

where W is the total weight of the designed structure; n is the number of all truss members; ρ_i is the material density of the i-th member for $\forall i \in \{1,\ldots,n\}$; A_i is the member cross-sectional area (defined as the design variables); L_i is the member length; σ_i is the member stress; σ_{all} is the allowable stress; δ_j is the nodal displacement for $\forall j \in \{1,\ldots,d\}$; and δ_{all} is the limited displacement at some j-th specified degree of freedom.

Turbulent Flow of Water-based Optimization

The TFWO is based on the whirlpool behaviors in developing the robust grouping optimization. The algorithm defines X_i as the position of the object and Wh_j the position of each whirlpool (i.e., being the best position occurring in the object). In the beginning, the method divides the population into various whirlpool sets, where the best position in each group generates the traction strength. Each whirlpool is unifying the object positions to the whirlpool center by applying the centripetal force (i.e., $X_i = Wh_j$). Other whirlpools lead to some deviations resulting in the new position of the object described by

$$\Delta X_i = \left(\cos(\delta_i^{new}) * rand\ (1, D) * \left(Wh_f - X_i\right) - \sin(\delta_i^{new}) * rand\ (1, D) * (Wh_w - X_i)\right) * \\ (1 + |\cos(\delta_i^{new}) * -\sin(\delta_i^{new})|) \tag{3}$$

$$X_i^{new} = Wh_j - \Delta X_i \tag{4}$$

At variance with the centripetal force attracting the moving object toward its whirlpool, the centrifugal force pushes the object away the center. In the instance when the centrifugal force overcomes the centripetal counterpart as defined in Eq. (5), the object position transfers to the new position. The centrifugal force FE_i is described in Eq. (6) if it is greater than the random values:

$$FE_i = ((\cos(\delta_i^{new}))^2 * (\sin(\delta_i^{new}))^2)^2 \tag{5}$$

$$X_{i,p} = X_p^{min} + rand * (X_p^{max} - X_p^{min}) \tag{6}$$

Moreover, the position of whirlpools can be influenced by the other whirlpool. The whirlpool positions is updated as follows:

$$Wh_j^{new} = Wh_f - \Delta Wh_j \tag{7}$$

$$\Delta Wh_j = rand\ (1, D) * \left| cos(\delta_j^{new}) + sin(\delta_j^{new}) * (Wh_f - Wh_j) \right| \tag{8}$$

In the case where the best object among all members in the set is stronger than the whirlpool itself, the new whirlpool is updated by this best object for the consequent iteration.

Illustrative Example

The applications of the proposed TFWO method have been illustrated through the sizing optimization of 3D steel trusses under strength and serviceability constraints. A number of design benchmarks and examples have been successfully solved by the present scheme. One of which, namely the design of 25-bar space tress in Fig. 1 [6] is considered in this work. The design forces applied were 0.5 kip at node 3 in the positive x-direction, 0.6 kip at node 6 in both negative y and z directions and 10 kips at nodes 1 and 2 in both negative y and z directions.

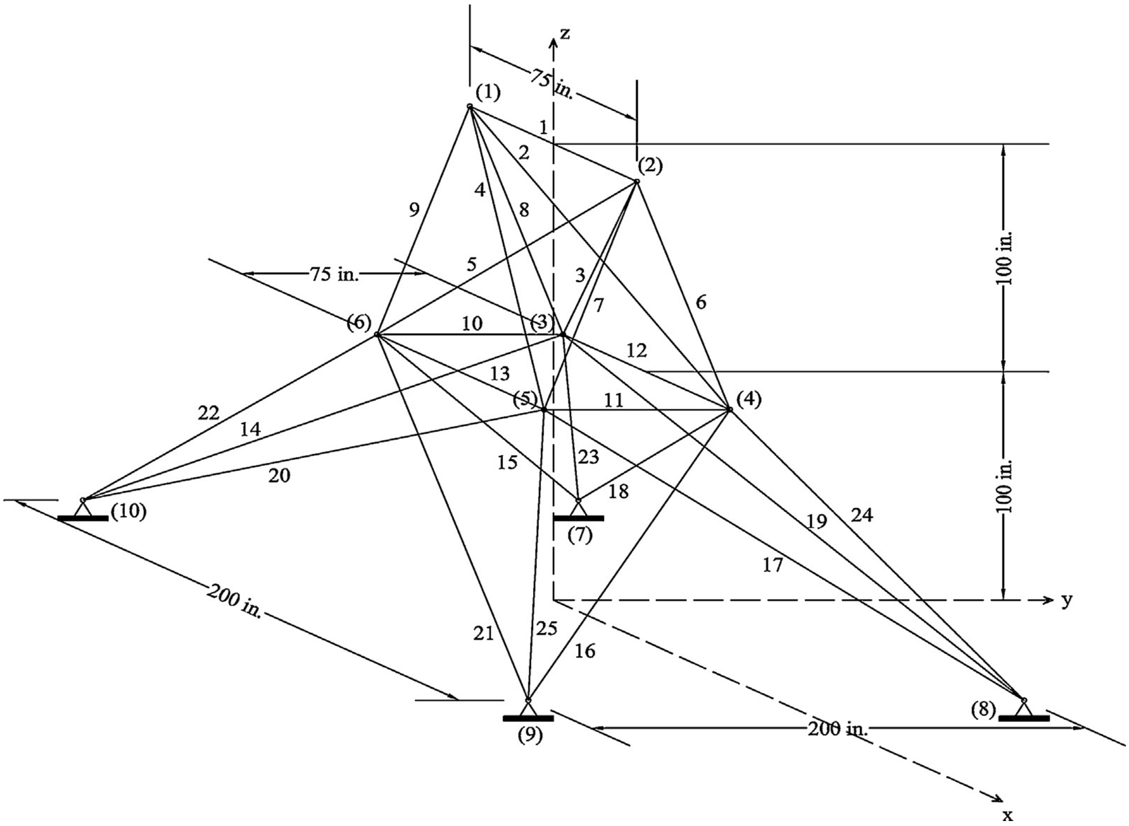

Fig. 1. 25-bar space truss structure

The design variables defined the member areas categorized into 8 different groups as listed in Table 1. The material properties employed were elastic modulus of 10,000 ksi (68,950 MPa) and uniform material density of 0.1 lb.in^{-3} (2767.99 kg.m^{-3}). The cross-sectional areas were selected within the range between 0.01 in^2 and 3.4 in^2. The allowable displacements of each node were limited to the variation of 0.35 in at x- and y-directions. The maximum stress limits in all compression and tension members are listed in Table 1.

Table 1 Member group and stress limits.

Group	Members	Compression stress limit (ksi)	Tension stress limit (ksi)
1	A_1	35.092	40
2	$A_2 – A_5$	11.59	40
3	$A_6 – A_9$	17.305	40
4	A_{10}, A_{11}	35.092	40
5	A_{12}, A_{13}	35.092	40
6	$A_{14} – A_{17}$	6.759	40
7	$A_{18} – A_{21}$	6.959	40
8	$A_{22} – A_{25}$	11.082	40

The optimal design of the steel space truss was successfully performed by the proposed TFWO method within 50 analysis iterations. The solution (total weight) convergence with the number of analysis (up to 400) iterations is clearly depicted in Fig. 2. More explicitly, the minimum weight of 482.026 lbs was computed at the 42-th iteration and took only 23 seconds. The optimal results, including the total weight and designed member areas, are reported in Table 2, and agree well with those from benchmarks [6], [7], [8], [9] ,[10], and [4]. In essence, the present TFWO approach provides the most minimum weight solution with the satisfaction of all constraints.

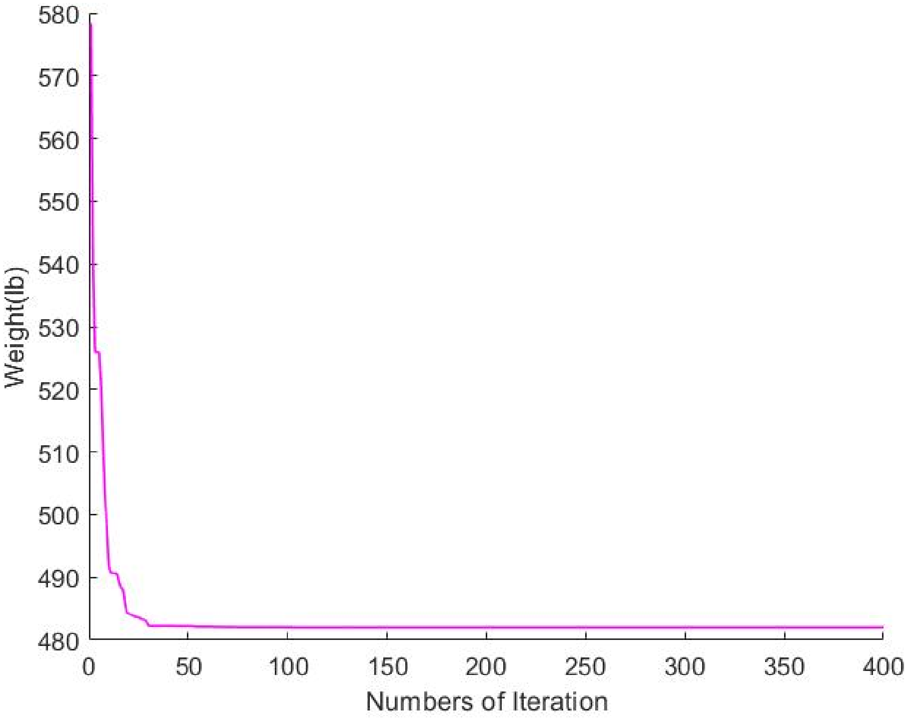

Fig. *2*. Convergence History of 25-bar space truss

Table 2 Optimum results for various design methods.

Design Variables	Cao et al. GA [8]	Li et al. HPSO [9]	TLBO [10]	Camp [4]	ACO [7]	FPA [6]	TFWO (Present)
A_1	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100
A_2	2.0119	1.9700	1.9878	2.0920	2.0000	1.8300	0.4231
A_3	2.9493	3.0160	2.9914	2.9640	2.9660	3.1834	3.4000
A_4	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100
A_5	0.0295	0.0100	0.0100	0.0100	0.0120	0.0100	1.9170
A_6	0.6838	0.6940	0.6828	0.6890	0.6890	0.7017	0.9653
A_7	1.6798	1.6810	1.6764	1.6010	1.6790	1.7266	0.4728
A_8	2.6759	2.6430	2.6656	2.6860	2.6680	2.5713	3.4000
Weight (lb)	545.8000	545.1900	545.1750	545.3800	545.5300	545.1590	482.0268

Concluding Remarks

The novel TFWO method has been presented for the optimal sizing design of steel space truss structures under applied forces. Both the material capacities and displacement limits are imposed directly as the design constraints. The TFWO determines the minimum of the total weight (cost) of the design structure with the strict satisfaction of all constraints. A number of design examples have been successfully proceeded by the proposed scheme at modest computing efforts. The accuracy of the optimal designed variables as compared to the benchmarks can be achieved. In essence, the more minimum weight of all members employed as illustrated in this paper is evidenced.

Acknowledgments

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