Numerical solution of a bioheat transfer problem with transient blood temperature

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Abstract

In the heat treatment process, blood perfusion starts up a negative feedback mechanism. The blood temperature undergoes a transient process before onset of equilibrium, and then changes the situation of temperature distribution. In substance, the blood temperature undergoes a transient process for heat exchange between blood and tissue. For more fully exploring the heat transfer behavior of biological tissue, this paper analyzes the bio-heat transfer problem with the non-constant blood temperature based on the Pennes bioheat equation. A numerical scheme based on the Laplace transform is proposed to solve the bio-heat transfer problem with simultaneous equations.

Keywords: Bio-heat transfer, blood temperature, Laplace transform, Pennes bioheat equation

Introduction

Based on experiment analysis, in1948, Pennes[1] proposed the first constitutive relationship between temperature and the blood flow rate. This relation is popularly known as Pennes' bioheat equation. The equation includes a special term that describes the heat exchange between blood flow and solid tissues. Many researchers used it to deal with various problems. The literature [2, 3] modeled small breast carcinomas surrounded by extended health tissue as a solid sphere and investigated the effect of dose on the temperature distribution. Kuznetsov[4] explored the temperature distribution with a transient thermal dose and investigated the effect of thermal dose accumulation during cooling. Michelea et al. [5] studied how the infusion behavior of magnetic nanofluids affects the thermal response in tissue. Lin et al. [6] numerically studied the bio-heat transfer problem in a bi-layered spherical tissue with blood perfusion and metabolism. Kudryashov and Shilnikov[7] used the Pennes bioheat model to describe he heat transfer in soft tissue during the thermal exposure to low temperature. Ma et al. [8] analyzed the effect of controlling the blood perfusion and temperature into the brain on brain hypothermia.
It is believed that even the applications with the estimated values do not affect explanation of the applicability of the bioheat transfer model [9]. For convenience of analysis, therefore, the above papers [2-8] regarded the blood temperature as a constant. In the heat treatment process, blood perfusion starts up a negative feedback mechanism. The blood temperature undergoes a transient process before onset of equilibrium, and then changes the situation of temperature distribution. In substance, the blood temperature undergoes a transient process for heat exchange between blood and tissue [10]. For more fully exploring the heat transfer behavior of biological tissue, this paper analyzes the bio-heat transfer problem with the non-constant blood temperature. A numerical scheme based on the Laplace transform is proposed to solve the present problem.

Problem Formulation

Energy conservation equation of bioheat transfer described in the Pennes model is

$$-\nabla \cdot \bar{q} + w_b c_b (T_b - T) + q_m + q_r = \rho c \frac{\partial T}{\partial t} \tag{1}$$

Here, ρ, c, and T denote density, specific heat, and temperature of tissue. c_b and w_b are, respectively, the specific heat and perfusion rate of blood. q_m is the metabolic heat generation and q_r is the heat source for spatial heating. T_b is the arterial temperature.

This work considers that the skin surface temperature could be kept constant as the skin contacts with a large steel plate at a high temperature. The assumption that heat flux approaches zero deep in tissue $x = L$ was made. The present work defines the heat transport in the skin with constant physiological parameters as the following equations.

$$\rho C \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} + W_b \rho_b C_b (T_b - T) + q_m + q_r \tag{2}$$

And then, the he boundary conditions can be written as

$$T(0,t) = T_o \quad \text{and} \quad \frac{\partial T(L,t)}{\partial x} = 0 \tag{3}$$

and the initial conditions

$$T(x,0) = T_{bi}\,, \ \frac{\partial T(x,0)}{\partial t} = 0\,, \text{ and } \ q(x,0) = 0 \tag{4}$$

where T_{bi} is the initial blood temperature and is specified as 37 ^{o}C.

The blood temperature always was assumed to be constant arterial blood temperature for studying such problems. In substance, the blood temperature undergoes a transient process for heat exchange between blood and tissue. The transient process was defined by [6]

$$\varepsilon \rho_b c_b \frac{\partial T_b}{\partial t} = G(T - T_b) \tag{5}$$

where G is the coupling factor between blood and tissue. ε is a proportional rate.

Analytical Method

Two new variables H and T_B are defined as $H = T - T_{bi}$ and $T_B = T_b - T_{bi}$. Eq. (2) can be rewritten for $q_r = 0$ as

$$\rho c \frac{\partial H}{\partial t} + w_b \rho_b c_b H = k \frac{\partial^2 H}{\partial x^2} + w_b \rho_b c_b T_B + q_m \tag{6}$$

The boundary conditions become

$$H(0,t) = H_0 \tag{7}$$

$$\frac{\partial H(L,t)}{\partial x} = 0 \tag{8}$$

The initial conditions are rewritten as

$$H(x,0)=0\,,\ \frac{\partial H(x,0)}{\partial t}=0\,,\text{ and }\ q(x,0)=0 \tag{9}$$

Subsequently, the use of the Laplace transform technique maps the transient problem into the steady one. The differential equations (5) and (6) and the boundary conditions (7) and (8) are transformed in conjunction with the initial conditions (9) as

$$\frac{d^2\tilde{H}}{dr^2}-\lambda^2\tilde{H}=-f \tag{10}$$

and

$$\tilde{H}(0,s)=H_0/s \tag{11}$$

$$\frac{d\tilde{H}(L,s)}{dx}=0 \tag{12}$$

where

$$\lambda^2=\frac{1}{k}[\rho cs+w_b\rho_b c_b(1-\frac{1}{1+\tau_p s})] \tag{13}$$

$$f=\frac{q_m}{ks} \tag{14}$$

$$\tau_p=\frac{\varepsilon\rho_b c_b}{G} \tag{15}$$

, and s is the Laplace transform parameter for time t.

The present work divides the whole space domain into several sub-space domains. For continuities of heat flux and temperature within the whole space domain, the following conditions are required at the interface of the sub-space domain j-1, $[x_{i-1},x_i]$, and the sub-space domain j, $[x_i,x_{i+1}]$.

$$\tilde{H}_{j-1}(x_i)=\tilde{H}_j(x_i) \qquad i=1,2,\ldots,n\ ;\ j=i \tag{16}$$

$$\frac{d\tilde{H}_{j-1}(x_i)}{dx}=\frac{d\tilde{H}_j(x_i)}{dx} \qquad i=1,2,\ldots,n\ ;\ j=i \tag{17}$$

where the subscript i is the number of node. n is the total number of nodes.

In order to perform the derivation of the governing algebraic equations, $\tilde{H}$ is approximated by using the nodal temperatures and shape function within a small sub-space domain. The shape function in each sub-space domain is derived from the governing equation (10) with the following procedures.

For the sub-space domain j, $[x_i,x_{i+1}]$, the analytical solution of the governing equation (10) subjected to the boundary conditions

$$\tilde{H}_j(x_i) = \tilde{H}_i \quad \text{and} \quad \tilde{H}_j(x_{i+1}) = \tilde{H}_{i+1} \tag{18}$$

are easily obtained and can be written as

$$\tilde{H}_j(x_i) = \tilde{H}_i \quad \text{and} \quad \tilde{H}_j(x_{i+1}) = \tilde{H}_{i+1} \tag{18}$$

$$\tilde{H}_j = \frac{1}{\sinh \lambda \ell}\left\{(\tilde{H}_i - \frac{f}{\lambda^2}x_i)\sinh \lambda(x_{i+1} - x) + (\tilde{H}_{i+1} - \frac{f}{\lambda^2}x_{i+1})\sinh \lambda(x - x_i)\right\} + \frac{f}{\lambda^2} \tag{19}$$

where ℓ denotes the length of sub-space domain or the distance between two neighboring nodes.

Substituting Eq. (16) and the shape function (19) into Eq. (17) and then evaluating the resulting derivative can lead to the discretized form for the interior nodes as following

$$\tilde{H}_{i-1} - 2\cosh(\lambda\ell)\tilde{H}_i + \tilde{H}_{i+1} = \frac{f}{\lambda^2}[2 - 2\cosh(\lambda\ell)] \tag{20}$$

Eq. (20) in conjunction with the discretized forms of the boundary conditions can be rearranged as the following matrix equation

$$[B]\{\tilde{H}\} = \{F\} \tag{21}$$

where $[B]$ is a matrix with complex numbers, $\{\tilde{H}\}$ is a column vector in the Laplace transform domain, and $\{F\}$ is a column vector representing the forcing term. Thereafter, the value of H in the physical domain can be determined with the application of the Gaussian elimination algorithm and the numerical inversion of the Laplace transform [11].

Results and Discussion

Some thermal properties of the sample skin are regarded as w_b = 0.5 kg/m^3· s, k = 0.2 W/m· ^{o}C, ρ = 1000 kg/m^3 and $c = c_b$ = 4200 J/kg· ^{o}C [11]. The distance between the skin and body core is L = 0.01208 m and the value of H_o is specified with 12 ^{o}C [11]. The values of the other parameters are individually determined for each calculation.

The primary premise of the Pennes bioheat equation is that the blood temperature is to be constant arterial blood temperature, and immediately equilibrates(thermally) with the surrounding tissue. In substance, the blood temperature undergoes a transient process before onset of equilibrium, so the assumption $\varepsilon\rho_b c_b \partial T_b/\partial t = G(T - T_b)$ was made [10]. Figure 1 shows the temperature variations at x = 0.01 m for the assumptions, $\varepsilon\rho_b c_b \partial T_b/\partial t = G(T - T_b)$ and T_b = 37^{0}C. It is observed that the cooling function of blood is reduced for the assumption $\varepsilon\rho_b c_b \partial T_b/\partial t = G(T - T_b)$, because the blood temperature would be increased from T_{bi} to the tissue temperature T [12]. The variation rates of temperature shown in Fig. 1(a) and Fig. 1(b) are obviously different. It expresses that the effect of location significantly affects the temperature variation at the measurement point.

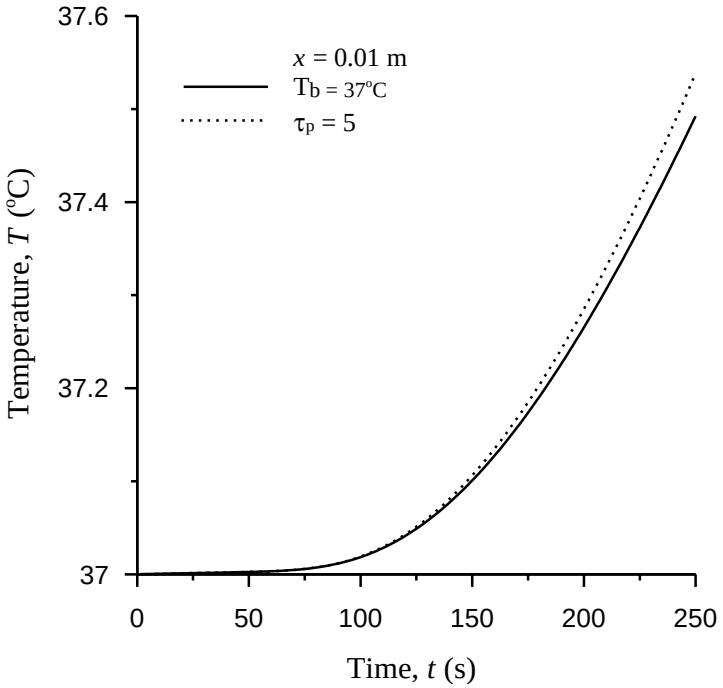

Figure 1. Temperature variations for the assumptions, $\varepsilon\rho_b C_b \partial T_b/\partial t = G(T - T_b)$ and $T_b = 37^0$C, at $x = 0.01$ m.

Figure 2 depicts the temperature variations at $x = 0.00208$ m for $\tau_p = 1$ and $\tau_p = 9$. It is observed from Fig. 1 that the temperature at $x = 0.00208$ m increases with the time. However, Ref. [11] indicated that as the blood temperature is specified as 37 °C, the temperature at $x = 0.00208$ almost has been in steady state after $t = 250$ *s*. This result implies that the cooling function of blood will be reduced as the blood temperature undergoes a transient process. In this case, the curves of temperature variation for $\tau_p = 1$ and $\tau_p = 9$ are coincident. It implies the effect of τ_p is not obvious in the present case.

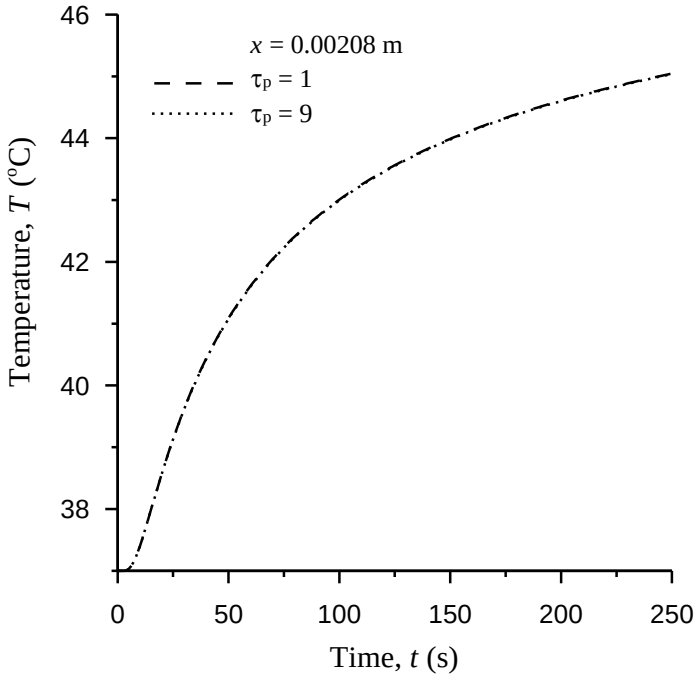

Figure 2. Temperature variations for $\tau_p = 1$ and $\tau_p = 9$ at $x = 0.00208$ m.

In order to further explore the effect of τ_p, the perfusion rate of blood is increased from w_b = 0.5 kg/m^3· s to w_b = 1.0 kg/m^3· s. Figure 3 shows the calculated results with w_b = 1.0 kg/m^3· s for τ_p = 1 and τ_p = 9. Two temperature variation curves are coincident. This phenomenon is same as that shown in Figure 2. It implies that the effect of τ_p is not strengthened with increasing the value of w_b in the present problem. The temperature distributions at t = 50 s and t = 150 s for τ_p = 1 and τ_p = 9 with w_b = 1.0 kg/m^3· s are also presented in Figure 4.

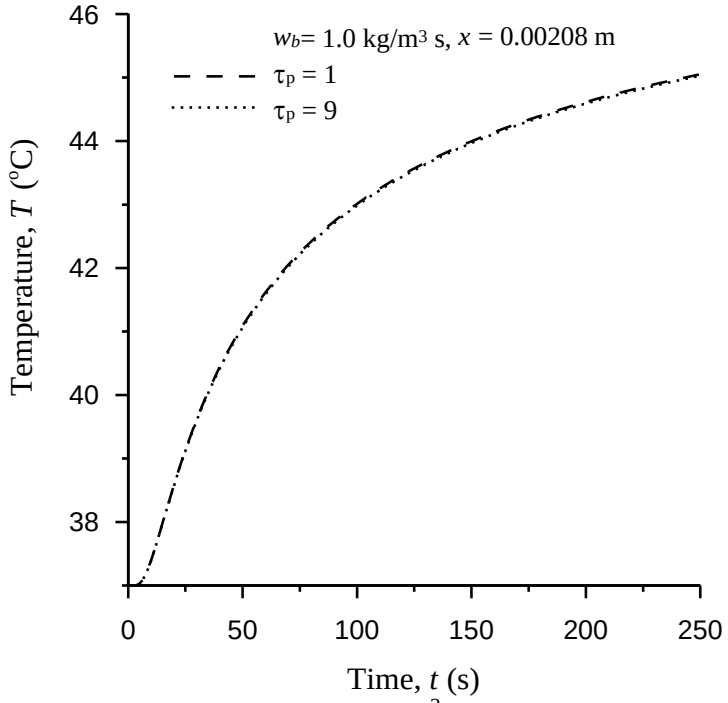

Figure 3. Temperature variations with w_b = 1.0 kg/m^3· s for τ_p = 1 and τ_p = 9 at x = 0.00208 m.

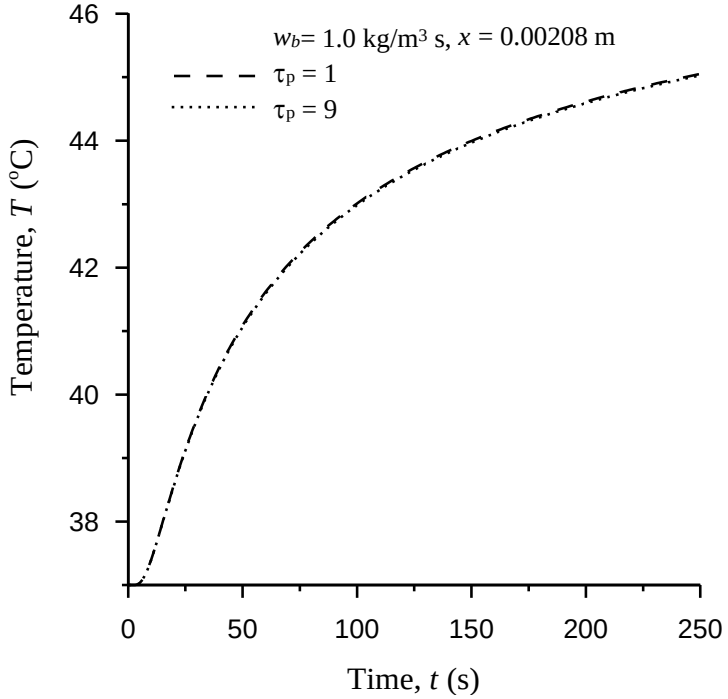

Figure 4. Temperature distributions at t = 50 s and t = 150 s for τ_p = 1 and τ_p = 9 with w_b = 1.0 kg/m^3· s.

Conclusions

A numerical scheme based on the Laplace Transform method is proposed for solving the Pennes bio-heat transfer equation with transient blood temperature. The results without constant blood temperature obviously differ from those with constant blood temperature. The effect of τ_p is not strengthened with increasing the value of w_b in the present problem. The present study depicts that the effects of τ_p and w_b are not obvious under that the blood temperature undergoes a transient process for heat exchange between blood and tissue.

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