A finite element method used for contact analysis of rolling bearings

†*S. Li[1] and M. Motooka[1]

[1]Interdisciplinary Graduate School of Science and Engineering, Shimane University, Japan

*Presenting author: shutingli@ecs.shimane-u.ac.jp
†Corresponding author: shutingli@ecs.shimane-u.ac.jp

Abstract

This paper deals with contact analysis method of rolling bearings. A mathematical model is presented for the purpose of contact analysis of rolling bearings based on the principle of mathematical programming method at the first. Then, three-dimensional (3D), finite element method (FEM) is introduced in the mathematical model to calculate deformation influence coefficients and gaps of assumed pairs of contact points between contact surfaces. Special software is developed to realize the procedures of contact analysis. With the help of the developed software, contact analyses are conducted for a deep groove ball bearing and a cylindrical roller bearing. In the case of the ball bearing, it is found that the calculated contact pressures on ball surface are more reasonable and accurate than the ones obtained by commercial CAE software. In the case of the roller bearing, it is found that edge-loads (non-Hertz contact that cannot be analyzed by Hertz theory) on the two ends of the roller surface are analyzed successfully by the method presented in this paper when the rollers are not crowned longitudinally. It is also found that the edge-loads are disappeared and the contact pressure becomes uniform distribution on the roller surface when the rollers are crowned on the two ends using Johson-Gohar [1] curve. Since the calculated results given in this paper cannot be obtained by using commercial CAE software and other numeric methods, the mathematical model and numeric method presented in this paper have a great practical meaning in engineering design and calculations of the rolling bearings.

Keywords : Finite element method, Contact analysis, Mathematical programing method, Rolling bearing

1. Introduction

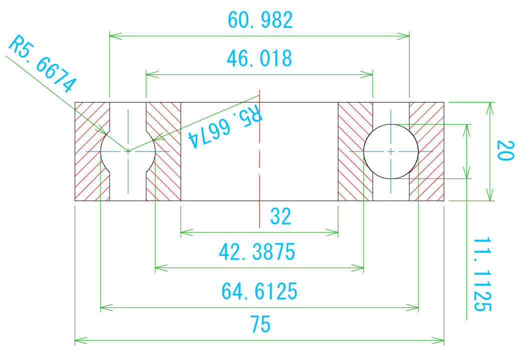
It is a very important thing for machine designers to evaluate lifetime and radial rigidity of the rolling bearings when they decide to use. Unfortunately, it is still a difficult thing to evaluate contact strength and lifetime of the rolling bearings accurately in theory. Also, it is still a difficult thing to calculate contact pressure and radial rigidity of the roller bearings accurately in theory. This is because there have been still some unsolved problems remained in strength and performance analyses of the rolling bearings, though it is a very long history to use the rolling bearings in various kinds of machines.

In the case of the ball bearings, usually, Hertz theory [2] is used to calculate the contact pressure and radial rigidity of the ball bearings. Since Hertz theory can only consider local deformation of contact areas of the ball bearings and the total structural deformation of the ball, outer ring and inner ring as well as housings cannot be included, Hertz theory has a limit in engineering calculations when the total structural deformation mentioned above is considered. In the case of the roller bearings, since edge-loads exist between the two contact surfaces, contact problem of the roller bearings belongs to a non-Hertz contact problem and Hertz theory cannot be used for contact analysis of the roller bearings. In order to solve the contact problem of the roller bearings, an approximate contact model of using a roller contacting a surface of infinite length was used [3]-[5]. But since the surface of infinite length also cannot consider the effects of structural sizes and shapes of the inner and outer rings, this contact model is also not so accurate for contact analysis of the roller bearings. Finally, FEM was suggested to do contact analysis of the roller bearings [3][6]. Indeed, FEM is a very practical method for structural analysis and very successful in many kinds of engineering calculations. But, unfortunately, this method is not so successful in contact analysis of machines and machine elements. The problem of using some commercial CAE software in contact analysis of the bearings shall be introduced in Section 3 of this paper.

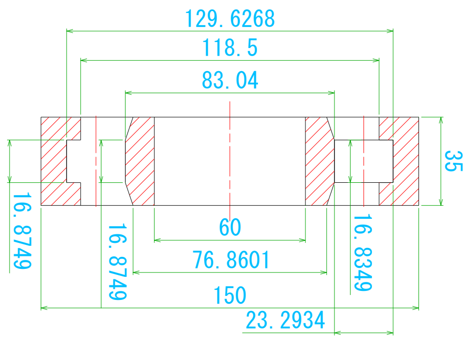
This paper tries to present a new FEM that can conduct contact analysis of the rolling bearings accurately. Firstly, a new mathematical model is presented in this paper for contact analysis of the rolling bearings based on the principle of the mathematical programming method. Then, 3D, FEM is introduced in the mathematical model to calculate deformation influence coefficients and gaps of the assumed pairs of contact points on the contact surfaces. Special software is developed through efforts of many years. With the help of the developed software, contact analyses of a deep groove ball bearing and a cylindrical roller bearing are conducted successfully. Calculation results shows that the special software can calculate more reasonable and accurate contact pressure distribution of the rolling bearings than the commercial software SolidWorks and some other finite element method [10] stated in this paper. The maximum contact pressure and radial contact rigidity of the ball bearing are also analyzed with Hertz theory. It is found that the results obtained by the special software are similar to the results obtained by Hertz theory, but they are not exactly equal. The total structural deformation of the ball, outer ring and inner ring can be thought to be the main reason to result in the difference between the method presented in this paper and Hertz theory. This assumption shall be confirmed experimentally in the near future.

2. Structural dimensions of the bearings used as research objects

Structures and dimensions of the rolling bearings used as research objects are illustrated in Fig.1. In Fig.1, (a) and (b) are a deep groove ball bearing (type number 6332) and a cylindrical roller bearing (type number NU412) respectively. They are made by NTN, a Japanese bearing company [8]. Contact analyses are conducted for them with commercial software SolidWorks and special FEM software developed in this paper respectively. Hertz theory is also used to calculate contact pressure and radial rigidity of the ball bearing in this paper in order to make a comparison with the results obtained by the special FEM software.



(a) Ball bearing



(b) Roller bearing

Figure 1. Structures of the ball and roller bearings used as research objects

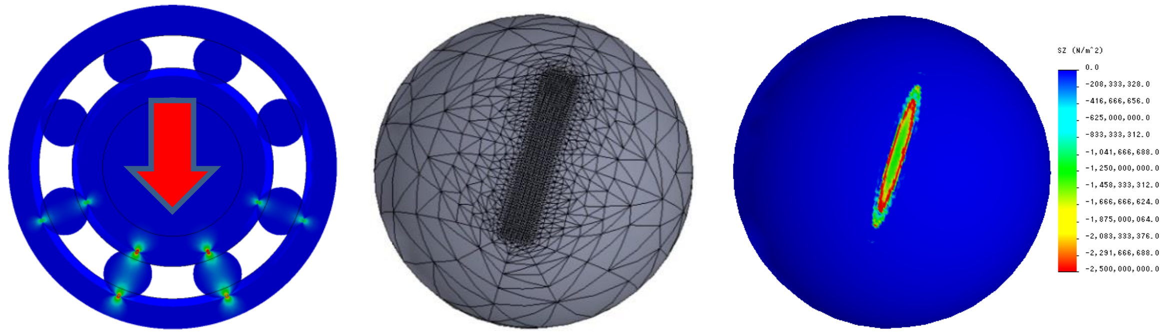
3. Problems of some commercial software used for contact analysis

As stated above, some commercial software is very successful in many kinds of engineering calculations, but it is not so successful in contact analysis of machines and machine elements. In order to make this problem clear in this paper, some results of using the commercial software SolidWorks for contact analysis of the ball and roller bearings are introduced in the following.

In SolidWorks software, there is a function called SolidWorks Simulation that can be used to do CAE simulations. SolidWorks Simulation is originated from the famous CAE software COSMOSWorks [9]. In the COSMOSWorks, contact analysis function is also included. So, this paper uses this function to conduct contact analyses of the ball and roller bearings given in Fig. 1. When the contact analyses are conducted, outside surfaces of the bearing outer rings are fixed as boundary conditions and a radial load P is applied on the inside surfaces of the bearing inner rings as shown in Fig. 2(a) and 3(a) through bearing shafts that are inserted into the central holes of the inner rings. The bearing shaft and the inner ring are unified as one elastic body in the analyses.

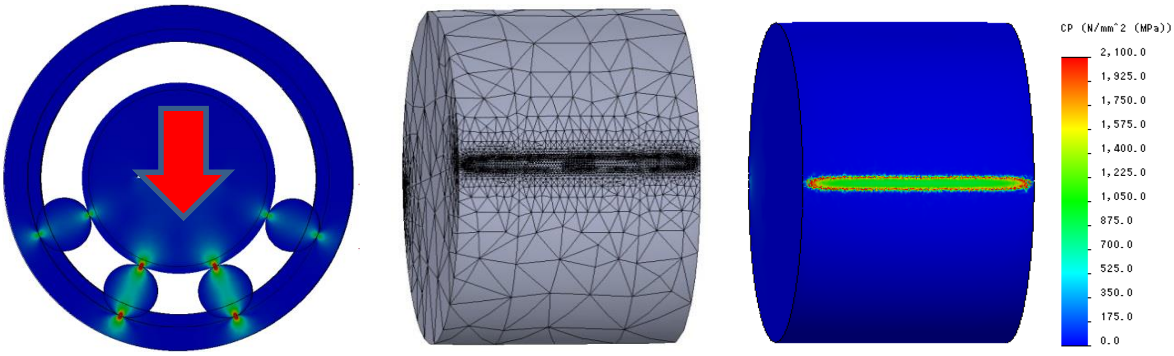
Calculation results of the ball bearing are given in Fig. 2. In Fig. 2, (a) is a contour map of calculated Von Mises stresses distributed on the section that goes through the ball center and is perpendicular to the bearing axis. Fig. 2(a) indicates that only four balls at the lower part of the bearing are in contact with the raceways of the inner and outer rings. Fig. 2(b) is FEM mesh-diving pattern of the ball. As shown in Fig. 2(b), contact areas of the ball surface are fine mesh-divided in order to ensure high calculation accuracy. Of course, the contact areas of the raceways of the outer and inner rings are also fine mesh-divided responsively. Fig. 2(c) is a contour map of calculated contact pressure distributed on the ball surface. From Fig. 2(c), it is found that though contact pattern of the ball takes the shape of an elliptical contact, the maximum contact pressure is not located at the center of the elliptical area. It distributes along a closed elliptical curve as shown in Fig .2(c) illustrated in the red line. Also the maximum contact pressure is calculated to be about twice the value calculated by Hertz theory. This means that SolidWorks cannot calculate contact pressure distribution and the maximum contact pressure correctly if it is used to conduct contact analysis of the ball bearings.

Calculation results of the roller bearing are given in Fig. 3. In Fig. 3, (a) is a contour map of calculated Von Mises stresses distributed on the section going through the center point of the roller width and being perpendicular to the bearing axis. Fig. 3(a) also indicates that only four rollers at the lower part of the bearing contact the raceways of the inner and outer rings. Fig. 3(b) is FEM mesh-diving pattern of the roller. As shown in Fig. 3(b), contact areas of the roller surface are fine mesh-divided. Of course, contact areas of the raceways are also fine mesh-divided responsively. Fig. 3(c) is a contour map of calculated contact pressure distributed on the roller surface. Fig. 3(c) indicates that contact pattern of the roller bearing takes the shape of an elliptical contact (the roller is crowned longitudinally using Johson-Gohar curve [1]), but the maximum contact pressure is also not located at the center of the contact area. It distributes along a closed elliptical curve as shown in Fig .3(c) illustrated in the red line. It means that there is also a problem existing for the roller bearing that SolidWorks cannot calculate contact pressure distribution of the roller bearing correctly. Per a long-time experience of the author on CAE analysis using commercial CAE software, it is found that not only SolidWorks, but also some other commercial software, such as ANSYS and ADINA, have the similar problem like SolidWorks that they cannot calculate contact pressure distribution accurately when they are used to do contact analyses of machines or machine elements.



(a) Von Mises stresses (b) FEM mesh-dividing (c) Contact pressure

Figure 2. Calculation results of the ball bearing using SOLIDWORKS

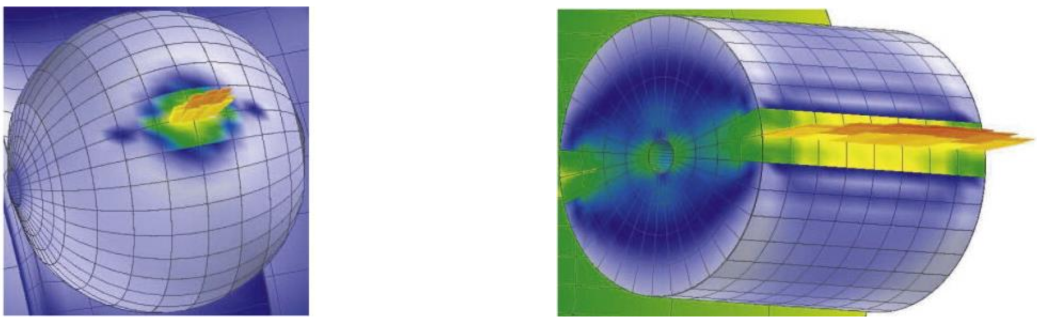


(a) Von Mises stresses (b) FEM Mesh-dividing (c) Contact pressure

Figure 3. Calculation results of the roller bearing using SOLIDWORKS

Guo and Parker [10] also conducted contact analyses for a deep groove ball bearing and a cylindrical roller bearing using FEM and specially developed software. Calculation results obtained by Guo and Parker are given in Fig. 4. Fig. 4(a) and (b) are calculated contact loads distributed on the ball and the roller surfaces respectively. From Fig. 4, it is found that quite rough results were obtained in Guo and Parker's research.

Based on the results mentioned above, it can be understood well that it is a quite difficult thing to conduct loaded bearing contact analysis and get correct contact pressure distribution of the bearings using available commercial CAE software and finite element techniques at the present situation. So, it is necessary to develop a new method and technology that can conduct contact analysis of the bearings correctly. This paper tries to present a new mathematical model and numeric method for contact analysis of the rolling bearings.

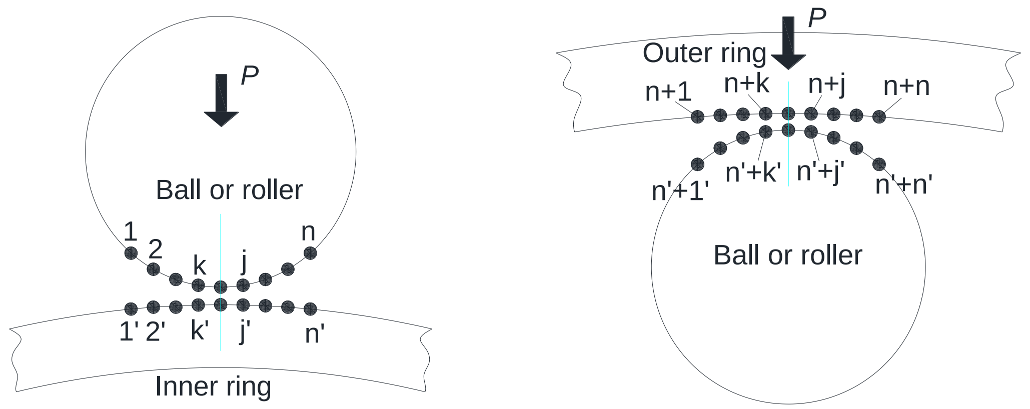


(a) Load distribution on ball surface (b) Load distribution on roller surface

Figure 4. Guo and Parker's results on bearing contact analysis [10]

4. A new mathematical model and numeric method for bearing contact analysis

4.1 Principle used for contact analysis of the rolling bearings



(a) Contact of ball with inner ring (b) Contact of ball with outer ring

Figure 5. Mathematical model used for contact analysis of rolling bearings

Models used for contact analysis of the ball and roller bearings are given in Fig. 5. In Fig. 5, (a) is used to stand for the contact of a ball (or roller) with the inner ring raceway and (b) is used to stand for the contact of the ball (or roller) with the outer ring raceway. It is assumed that an external load P (usually, equals to radial load of the bearings) is applied on the bearings in vertical direction as shown in Fig. 5. It is assumed that only elastic deformation occurred in the contact problem of the rolling bearings.

In Fig. 5(a), firstly, a lot of pairs of contact points, such as (1-1'), (2-2'), …, (j-j'), …, (k-k') and (n-n'), are assumed on the contact surfaces of the ball (roller) and the inner ring raceway along the vertical direction. In Fig. 5(a), 1,2, …, j, …, k and n are the assumed points on the contact area of the ball (roller) surface and 1',2', …, j', …, k' and n' are the responsive points on the contact area of the inner ring raceway. The common normal lines of these assumed pairs of contact points are parallel to the vertical direction and pass through the pairs of contact points. It is assumed that these pairs of contact points have possibility to come into contact when the external load P is applied.

4.1.1 Deformation compatibility relationship of the pairs of contact points

As shown in Fig. 5(a), for an optional pair of contact points (k-k'), F_k is used to denote the contact force between the pair of contact points (k-k'). Of course, direction of F_k is along the direction of its common normal line. Also, F_j is the contact force between the pair of contact points (j-j') along its common normal line. Gaps between the pairs (j-j'), (k-k') and (n-n') are denoted as ε_j, ε_k and ε_n respectively. Relative deformation of the ball (roller) relative to the inner ring along the vertical direction is denoted as δ_1. Elastic deformation of the pair of contact points (k-k') along its common normal line direction are denoted as ω_k and $\omega_{k'}$ respectively. If (k-k') comes into contact after P is applied, $(\omega_k + \omega_{k'} + \varepsilon_k)$, the amount of the deformation and the gap of the pair of points (k-k'), shall be equal to the relative deformation δ_1. But, if (k-k') doesn't come into contact, $(\omega_k + \omega_{k'} + \varepsilon_k)$ shall be greater than δ_1. These relationships are called deformation compatibility relationships and they can be expressed with Eq. (1) and (2) in the following. Eq. (1) and (2) can be summarized into Eq. (3).

$$\omega_k + \omega_{k'} + \varepsilon_k - \delta_1 > 0 \quad \text{(Not contact)} \tag{1}$$

$$\omega_k + \omega_{k'} + \varepsilon_k - \delta_1 = 0 \quad \text{(Contact)} \tag{2}$$

$$\omega_k + \omega_{k'} + \varepsilon_k - \delta_1 \geq 0 \qquad (k = 1, 2, \dots, n) \tag{3}$$

Eq. (3) is not only suitable for an optional pair of contact points (k-k'), but also suitable for all the pairs of contact points assumed on the contact surfaces of the ball (roller) with the inner ring raceway. In Eq. (3), n is the total number of the assumed pairs of contact points.
Since the elastic deformation ω_k and $\omega_{k'}$ can be expressed with deformation influence coefficients a_{kj} and $a_{k'j'}$, then Eq. (4) and (5) can be obtained. If Eq. (4) and (5) are substituted into Eq. (3), then Eq. (6) can be obtained.

$$\omega_k = \sum_{j=1}^{n} a_{kj} F_j \tag{4}$$

$$\omega_{k'} = \sum_{j=1}^{n} a_{k'j'} F_j \tag{5}$$

$$\sum_{j=1}^{n} \left[a_{kj} + a_{k'j'}\right] \times F_j + \varepsilon_k - \delta_1 \geq 0 \qquad (k = 1, 2, \dots, n) \tag{6}$$

Where, a_{kj} and $a_{k'j'}$ are deformation influence coefficients of the pairs of contact points along their common normal lines. a_{kj} and $a_{k'j'}$ can be calculated through 3D, finite element analysis.

4.1.2 Load equilibrium relationship of the pairs of contact points

Except for the deformation compatibility relationship as shown in Eq. (6), a load equilibrium relationship of the pairs of contact points can also be built as given in Eq. (7). Where, P is the external load applied on the bearing.

$$\sum_{k=1}^{n} F_k = P \qquad (k = 1, 2, \dots, n) \tag{7}$$

In the case of the ball (roller) contacting the outer ring raceway as illustrated in Fig. 5(b), the deformation compatibility relationship and load equilibrium relationship can also be built for the assumed pairs of contact points on the contact surfaces of the ball (roller) with the outer ring raceway in the same way. Eq. (8) and (9) are the two relationships for the pairs of contact points on the contact surfaces of the ball (roller) with the outer ring raceway.

$$\sum_{j=n+1}^{n+n} \left[a_{kj} + a_{k'j'}\right] \times F_j + \varepsilon_k - \delta_2 \geq 0 \qquad (k = n+1, \dots, n+n) \tag{8}$$

$$\sum_{k=n+1}^{n+n} F_k = P \qquad (k = n+1, \dots, n+n) \tag{9}$$

Where, δ_2 is the relative deformation of the ball (roller) relative to the outer ring along the vertical direction. By adding Eq. (6) and Eq. (8) together, then Eq. (10) can be obtained. Also, by adding Eq. (7) and Eq. (9) together, then Eq. (11) can be obtained. Where, $\delta = \delta_1 + \delta_2$ is the total relative deformation among the outer ring, the ball (roller) and the inner ring along the vertical direction.

$$\sum_{j=1}^{2n} \left[a_{kj} + a_{k'j'}\right] \times F_j + \varepsilon_k - \delta \geq 0 \qquad (k = 1,2, \dots, 2n) \tag{10}$$

$$\sum_{k=1}^{2n} F_k = 2P \qquad (k = 1,2, \dots, 2n) \tag{11}$$

If Eq. (10) is written into a matrix expression, then Eq. (12) can be obtained.

$$[S]\{F\} + \{\varepsilon\} - \delta\{e\} \geq \{0\} \tag{12}$$

Where,

$[\mathrm{S}] = \begin{bmatrix} [S1] & [0] \\ [0] & [S2] \end{bmatrix}$

$[S1] = \left[S_{kj}\right] = \left[a_{kj} + a_{k'j'}\right]$, $k = 1, 2, 3, \dots, n; j = 1, 2, 3, \dots, n$

$[S2] = \left[S_{kj}\right] = \left[a_{kj} + a_{k'j'}\right]$, $k = n+1, n+2, \dots, n+n; j = n+1, n+2, \dots, n+n$

$[0] = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}$

$\{F\} = \{F_1, F_2, \dots, F_k, \dots, F_{n+n}\}^T$

$\{\varepsilon\} = \{\varepsilon_1, \varepsilon_2, \dots, \varepsilon_k, \dots, \varepsilon_{n+n}\}^T$

$\{e\} = \{1, 1, \dots, 1\}^T$

$\{0\} = \{0, 0, \dots, 0\}^T$

Also, If Eq. (11) is written into a matrix form, Eq. (13) can be obtained.

$$\{e\}^T\{F\} = 2P \tag{13}$$

Eq. (12) and (13) can be used as constrain conditions in contact analysis of the bearings to identify which pair of contact points is in contact and which pair is not in contact when the external load *P* is applied. Contact problem of bearings can be explained as looking for the contact force F_k $(k = 1,2,3, \dots, 2n)$ of the pairs of contact points that must satisfy Eq. (12) and (13) under the conditions of knowing the deformation influence coefficients a_{kj}, $a_{k'j'}$, the gaps ε_k and the external load *P* in advance.

4.2. A new mathematical model used for contact Analysis of the rolling bearings

A new mathematical model is built to solve Eq. (12) and Eq. (13) based on the principle of the mathematical programming method [11]-[12] as follows. Since Eq. (12) is an inequality constraint equation that may be strictly positive or identically zero, it can be transformed into an equality constraint equation by introducing a so-called slack variable $\{Y\}$ (consists of positive variables) based on the principle of the modified simplex method [11]-[12]. Then Eq. (14) and (15) can be obtained.

$$[S]\{F\} + \{\varepsilon\} - \delta\{e\} - [I]\{Y\} = \{0\} \tag{14}$$

or

$$-[S]\{F\} + \delta\{e\} + [I]\{Y\} = \{\varepsilon\} \tag{15}$$

Where
$\{Y\} = \{Y_1, Y_2, \dots, Y_k, \dots, Y_{2n}\}^T$ (Slack variables)
$[I]$= a unit matrix of 2n×2n

Then the two equality constraint equations of Eq. (13) and (15) are obtained. The next task is to make an objective function Z that is necessary to build the mathematical programming model. The objective function Z can be made artificially through introducing some positive variables X_{2n+1}, X_{2n+2}, ..., X_{2n+2n}, $X_{2n+2n+1}$ (usually called artificial variables) to every constrain equation based on the principle of the modified simplex method [11]-[12]. Then the mathematical programming model used for contact analysis of the bearings can be made as follows.

Mathematical programming model used for bearing contact analysis
Objective Function:

$$Z = X_{2n+1} + X_{2n+2} + \cdots + X_{2n+2n} + X_{2n+2n+1} \tag{16}$$

Constraint Conditions:

$$-[S]\{F\} + \delta\{e\} + [I]\{Y\} + [I]\{Z'\} = \{\varepsilon\} \tag{17}$$

$$\{e\}^T\{F\} + X_{2n+2n+1} = 2P \tag{18}$$

Where,
$\{Z'\} = \{X_{2n+1}, X_{2n+2}, \dots, X_{2n+2n}\}^T$ (Artificial variables)

$[\mathrm{S}] = \begin{bmatrix} [S1] & [0] \\ [0] & [S2] \end{bmatrix}$

$[S1] = [S_{kj}]$, $k = 1, 2, 3, \dots, n; j = 1, 2, 3, \dots, n$

$[S2] = [S_{kj}]$, $k = n+1, n+2, \dots, n+n; j = n+1, n+2, \dots, n+n$

$[0] = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}$ $(n \times n)$

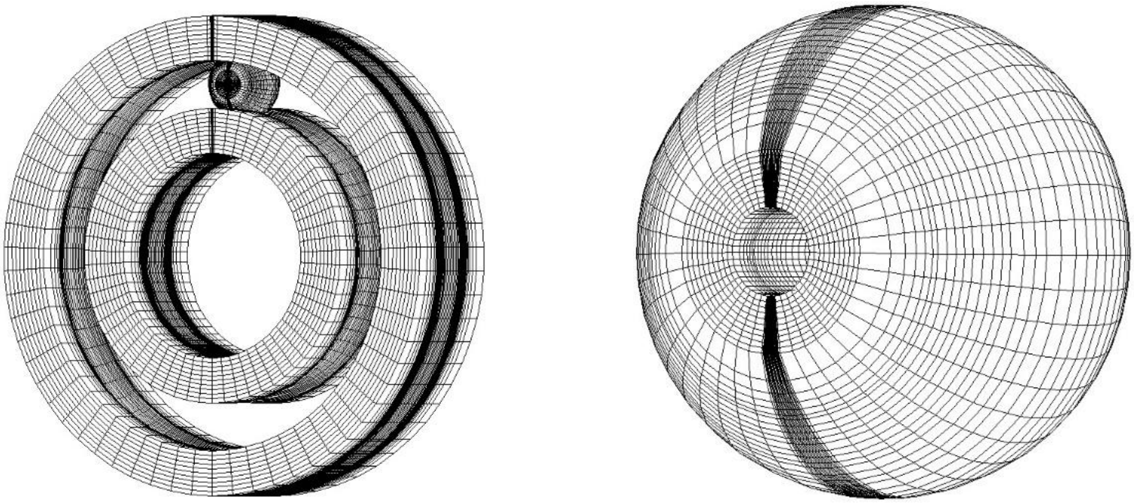
$\{F\} = \{F_1, F_2, \dots, F_k, \dots, F_{2n}\}^T$
$\{Y\} = \{Y_1, Y_2, \dots, Y_k, \dots, Y_{2n}\}^T$ (Slack variable)
$\{\varepsilon\} = \{\varepsilon_1, \varepsilon_2, \dots, \varepsilon_k, \dots, \varepsilon_{2n}\}^T$
$\{e\} = \{1, 1, \dots, 1\}^T$
$F_k \geq 0, Y_k \geq 0, \theta_k \geq 0, \theta \geq 0, k = 1, 2, \dots, 2n$
$X_{2n+m} \geq 0, m = 1, 2, \dots, 2n+1$

The contact force F_k and the total radial deformation δ can be calculated by minimizing the objective function Z in Eq. (16) under the constrain conditions of Eq. (17) and (18) using the modified simplex method [11]-[12].

4.3 Software development

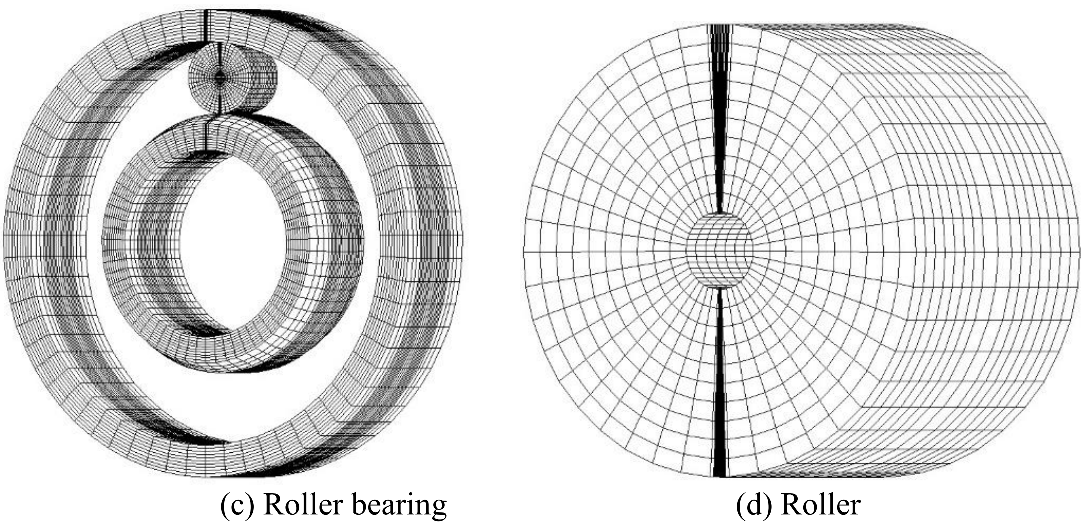
Software development is conducted to realize procedures of the bearing contact analysis. Firstly, 3D, FEM method is used to calculate the deformation influence coefficients a_{kj},

$a_{k'j'}$ that are necessary to form the [S1] and [S2] in the matrix [S]. Special FEM software is developed using Super-parametric hexahedron solid element, which has 8 nodes at the corner and 3 nodes inside the element [13]. FEM models and mesh-dividing patterns of the ball and roller bearings are given in Fig. 6 and 7 respectively. Fig. 6(a) and Fig. 7(a) are FEM models and mesh-dividing patterns of the whole ball and roller bearings respectively. Fig. 6(b) and Fig. 7(b) are enlarged views of the mesh-dividing patterns of the ball and roller only respectively. As shown in Fig. 6 and Fig. 7, meshes on the contact areas of the outer rings, ball (roller) and inner rings are fine divided in order to ensure high calculation accuracy of FEA.



(a) Ball bearing (b) Ball

Figure 6. Mesh-dividing patterns of the ball bearing



(c) Roller bearing (d) Roller

Figure 7. Mesh-dividing patterns of the roller bearing

Special software development is also conducted to realize the procedures of the mathematical programming for the bearing contact analysis after the deformation influence coefficients are available by FEA. Then, contact load $\{F\}$ and the total radial deformation δ can be available after the mathematical programming is conducted with the help of the developed software. Contact pressure distribution can be calculated after the contact load $\{F\}$ is available through calculating the contact load distributed on unit contact area. Also, radial rigidity K of the bearings can be calculated through this expression $K = P/\delta$. Calculation results are introduced in the following.

5. Calculation results and discussions

5.1 Contact pressure distribution

Firstly, loaded bearing contact analysis is conducted for the deep groove ball bearing as shown in Fig. 1(a) with the developed FEM software. Fig. 8(a) and 9(b) are calculated contact pressure distributed on the ball surfaces. Fig. 8(a) is a contour map of the contact pressure between the ball and the outer ring raceway. Fig. 8(b) is a contour map of the contact pressure between the ball and the inner ring raceway. The external load P is equal to 40kN when the contact analysis is conducted. From Fig. 8, it is found that the contact pressure on the ball surfaces is calculated to be beautiful elliptical distribution and the maximum contact pressure point is located at the center of the contact areas. These results are more reasonable than the results obtained by SolidWorks and the reference [10] as given in Fig. 2 and Fig. 4(a). It is also found that the maximum contact pressure on the upper part of the contact surfaces (the ball with the outer ring raceway) is a little smaller than the one on the lower part of the contact surfaces (the ball with the inner ring raceway). This is because the radius of curvature of the inner ring raceway is smaller than that of the outer ring raceway. The smaller radius of curvature of the contact surface shall bring greater contact stress based on Hertz theory.

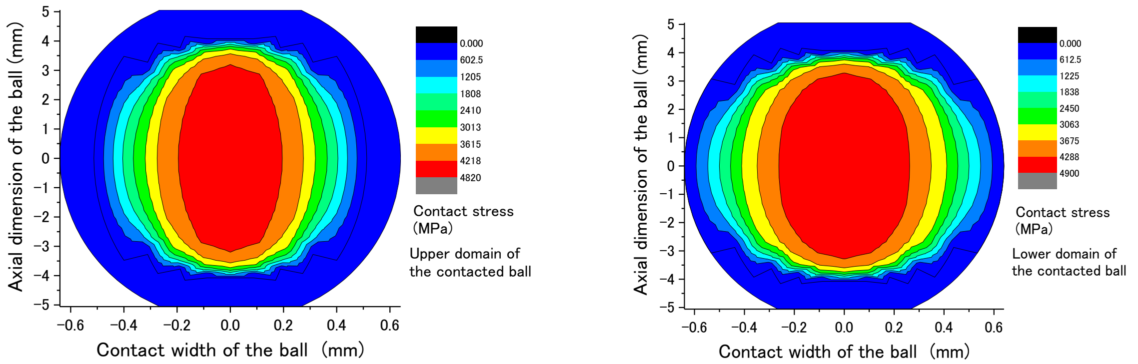
The maximum contact pressure of the ball bearing is also calculated with Hertz theory. Fig. 9 is a comparison of the maximum contact pressure between the developed FEM software and Hertz theory. In Fig. 9, abscissas are radial load P applied on the bearing and the ordinates are the maximum contact pressure on the ball surface. Fig. 9(a) is the maximum contact pressure between the ball and the inner ring raceway and Fig. 9(b) is the one between the ball and the outer ring raceway. Fig. 9(a) indicates that the results obtained by the FEM software are smaller than the ones obtained by Hertz theory. Fig. 9(b) indicates that the results obtained by the FEM software are greater than the ones obtained by Hertz theory. The difference between the two methods can be thought to be the effect of the total structural deformation of the bearing. As it has been stated above, Hertz theory cannot consider of the total structural deformation of the ball, inner and outer ring while FEM can consider the total structural deformation of the bearing. Secondly, loaded bearing contact analysis is conducted for the cylindrical roller bearing as shown in Fig. 1(b) with the developed FEM software. Fig. 10(a) and 10(b) are calculated contact pressure distributed on the roller surfaces when the roller is not crowned. Fig. 10(a) is the contact pressure between the roller and the outer ring raceway. Fig. 10(b) is the contact pressure between the roller and the inner ring raceway. The external load P is equal to 4kN when the contact analysis is conducted. From Fig. 10, it is found that contact pressure on the roller surface is calculated to be uniform distribution along axial direction of the roller except for the two end areas of the roller. It is also found that the edge-loads are calculated on the two end areas of the roller beautifully. It is a big success or progress that the developed FEM software can analyse edge-loads of an uncrowned roller bearing successfully. By comparing Fig. 10 with Fig. 4(b), it is found that the mathematical model and numeric method presented in this paper can calculate more reasonable results than method given in reference [10]. The fact is that it is still a difficult thing for some commercial CAE software to analyse the edge-loads correctly at the present situation.

Loaded bearing contact analysis is conducted also for the roller bearing when the roller is crowned on the two end areas using Johson-Gohar curve [1] as given in Eq. (19). Fig. 11(a) and 11(b) are imagines of the roller before and after crowned. Calculation results for the crowned roller bearing are given in Fig. 12. Fig. 12(a) is the contact pressure distribution between the crowned roller and the outer ring raceway. Fig. 12(b) is the contact pressure distribution between the crowned roller and the inner ring raceway. From Fig. 12, it is found that the edge-loads disappeared on the two end areas of the roller and contact pressure becomes uniform distribution longitudinally in comparison with the results given in Fig. 10. It is also found that the maximum contact pressures are reduced about 17% and 21% when the roller is crowned by comparing Fig. 12(a) with Fig. 10(a) and Fig. 12(b) with Fig. 10(b). The results in Fig. 12 indicate that Johson-Gohar curve is a very nice curve to be used as crowning curve for the roller bearings. It can reduce edge-loads greatly and bring the roller bearing a uniform contact pressure distribution.

$$q(x) = \frac{2P}{\pi l E'} ln \frac{1}{1-(1-0.3033b/a)(2x/l)^2} \tag{19}$$

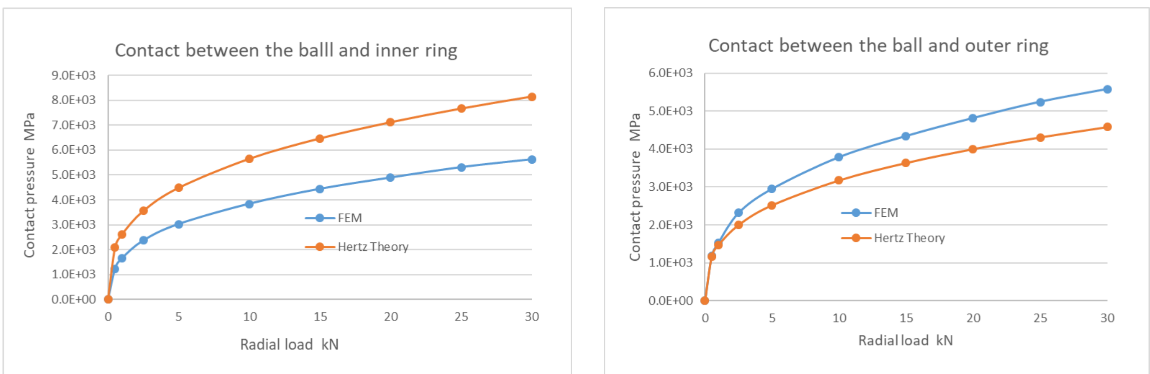
Where, l is an effective contact length of the roller and a is a half of the effective contact length l. b is a half width of the contact. E is Young's modulus and ν is Poisson's ratio. E′ is equivalent Young's modulus that can be obtained by following Eq. (20). P is a load applied on the roller. x is used to stand for longitudinal position of a point along the axial direction. $q(x)$ is used to denote the drop (quantity of crowning) at the position x in the axial direction.

$$\mathrm{E}' = \frac{E}{1-\nu^2} \tag{20}$$



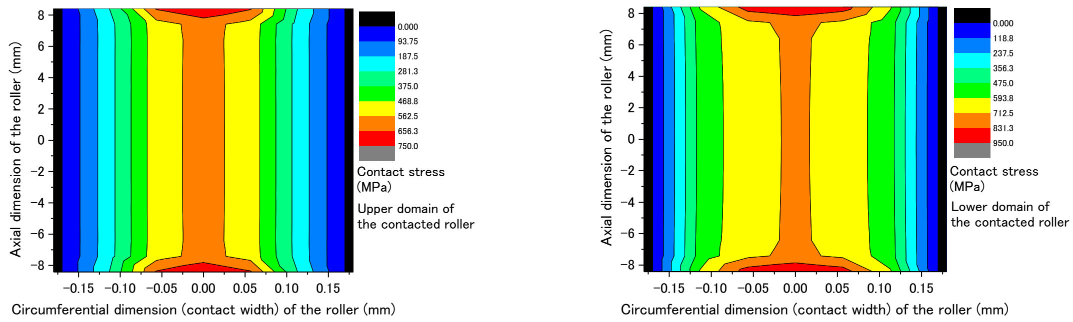
(a) The upper part of the contact domain (b) The lower part of the contact domain

Figure 8. Contour maps of contact stresses distributed on the ball surface



(a) Ball and inner ring (b) Ball and outer ring

Figure 9. Contact pressure comparison between FEM software and Hertz theory

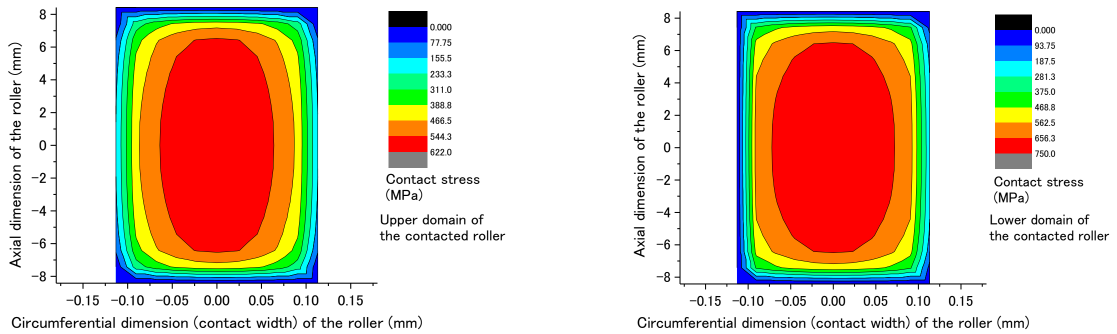


(a) The upper part of the contact domain (b) The lower part of the contact domain

Figure 10. Contour maps of contact stresses distributed on the roller surface

(a) The roller before crowned (b) The roller after crowned

Figure 11. Crowning on the two ends of the roller with Johson-Gohar curve



(a) The upper part of the contact domain (b) The lower part of the contact domain

Figure 12. Contour maps of contact stresses distributed on the roller surface

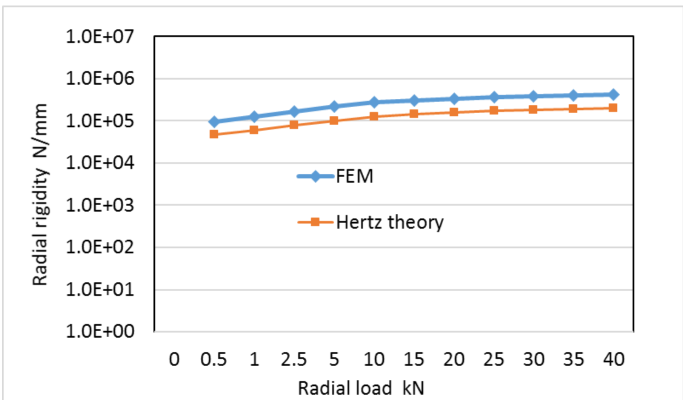


Figure 13. Comparison of radial rigidity of the ball bearing

5.2 Radial rigidity of the ball bearing

In the case of the ball bearings, since Hertz theory can be used to calculate radial rigidity of the bearings, a comparison of the radial rigidity is made for the ball bearing between the developed FEM software and Hertz theory in Fig. 13. From Fig. 13, it is found that the FEM results are a little greater than the Hertz theory results. An experimental research is scheduled to identify which method is more reasonable and accurately in the near future.

6. Conclusions

A mathematical model and numeric method are presented in this paper in order to conduct contact analysis of rolling bearings based on the principle of the mathematical programming method. Three-dimensional, finite element method is introduced to calculate deformation influence coefficients and gaps of the assumed pairs of contact points between contact surfaces. Special software is developed to realize the procedures of the contact analysis. With the help of the special software, loaded bearing contact analyses are conducted for a deep groove ball bearing and a cylindrical roller bearing. Calculation results shows that the special software can calculate more reasonable and accurate contact pressure distribution of the rolling bearings than the commercial software SolidWorks and some other methods. The maximum contact pressure and radial contact rigidity of the ball bearing are also analyzed with Hertz theory. It is found that the results obtained by Hertz theory are similar to the results obtained by the special software, but they are not equal exactly. An experimental research is scheduled to identify which method is more reasonable and accurately in the near future.

Acknowledgment

NSK Foundation for the Advancement of Mechatronics is appreciated for funding this research. Mr. Ryuichi SONEZAKI, a student who graduated from Machine Design Lab in Shimane University, is also appreciated for conducting contact analysis of the ball and roller bearings with SolidWorks software under the direction of the first author.

References

[1] John, P. M. and Gohar, R. (1981) Roller bearings under radial and eccentric loads, Tribology International, 14, 131-136.

[2] Tedric, A. Harris, Michael, N. Kotzalas (2007) Essential Concept of Bearing Technology: Rolling Bearing Analysis, Fifth Edition, CRC Press.

[3] Tedric, A. Harris, Michael, N. Kotzalas, (2007) Advanced Concepts of Bearing Technology: Rolling Bearing Analysis, Fifth Edition, CRC Press.

[4] Kamamoto S., Fujimoto K. and Yamamoto T. (2001) Research on crowning profile to obtain maximum load carrying capacity for roller bearings, KOYO Engineering Journal English Edition, 159, 47-52.

[5] Nagatani, H. (2016) A new resolution to contact problem of roller bearings (in Japanese), The Tribology, 6(346), 44-46.

[6] Zhao, H. (1998) Analysis of load distributions within solid and hollow roller bearings, Trans. ASME, J. Tribology 120(1), 134-139.

[7] Ryuichi SONEZAKI, R. and LI, S. (2014) Contact analysis and stiffness testing device design of rolling bearings, JSME Mechanical Engineering Congress, 1-4.

[8] Product catalogue of NTN Corporation

[9] Dassault Systemes SolidWorks Corporation, SolidWords Essentials

[10] Guo Y. and Parker R. G. (2012) Stiffness matrix calculation of rolling element bearings using a finite element/contact mechanics model, Mechanism and Machine Theory, 51, 32–45.

[11] Wolfe, P. (1959) The Simplex method for quadratic programming, Econometrica, 27, 382-398.

[12] Hiramoto, I. and Hase, A. (1973) Linear Programming Method, Baifukan Co., LTD. Press.

[13] Liu G. (1994), Structural Dynamics of the Finite Element Method (In Chinese), National Defense Industry Press.