SMOOTHED POLYHEDRAL VARIABLE-NODE ELEMENTS AND THEIR APPLICATIONS

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Abstract

The variable-node finite elements (VNEs) are very useful in dealing with mesh discontinuities under evolution in adaptive mesh refinement [1], as well as in mechanics problems involving physical discontinuities [2-4]. In this paper, a new type of variable-node elements that are allowed to take the arbitrary shapes of polyhedra with an arbitrary number of nodes and faces are constructed with the aid of the smoothed FEM (S-FEM), and their outstanding performance in terms of the accuracy and convergence rate is demonstrated through some numerical examples in contact mechanics.

Keywords: variable-node elements, smoothed FEM, contact mechanics, adaptive mesh refinement

Formulation of smoothed polyhedral variable-node finite elements

In order to construct the shape functions, the polyhedral element is divided into sub-tetrahedrons and the implicit shape functions are constructed based on the linear point interpolation. In the S-FEM based polyhedral element, the domain is discretized as in the conventional finite element, but the integration is conducted by means of the gradient smoothing technique [5]. In this integration, additional subdomains, called the smoothing domains, are built and the element performance and accuracy is up to how to construct these smoothing domains, like the NS-FEM (Node-based Smoothed FEM), the ES-FEM (Edge-based Smoothed FEM) and the CS-FEM (Cell-based Smoothed FEM) [5]. By the gradient smoothing technique, the displacement gradient $\nabla\mathbf{u}$ is smoothed, on smoothing domain Ω_k^s, as $\bar{\nabla}\mathbf{u}(\mathbf{x}_k) = \int_{\Omega_k^s} \hat{W}(\mathbf{x}_k - \mathbf{x})\nabla\mathbf{u}(\mathbf{x})\,d\Omega$, where $\hat{W}(\mathbf{x}_k - \mathbf{x})$ is a smoothing function associated with a typical point $\mathbf{x}_k$ of Ω_k^s, defined as $\hat{W}(\mathbf{x}_k - \mathbf{x}) = 1/V_k^s$ for $\mathbf{x} \in \Omega_k^s$ or $\hat{W}(\mathbf{x}_k - \mathbf{x}) = 0$ for $\mathbf{x} \notin \Omega_k^s$ with V_k^s being the volume of Ω_k^s. The smoothed displacement gradient $\bar{\nabla}\mathbf{u}$ is written in the matrix form in the nodal displacement column vector $\mathbf{d}_I$ as $\bar{\nabla}\mathbf{u}(\mathbf{x}_k) = \sum_{I=1}^{N_n^k} \bar{\boldsymbol{\beta}}_I(\mathbf{x}_k)\mathbf{d}_I^T$ utilizing the divergence theorem, where N_n^k is the total number of nodes associated with the smoothing domain Ω_k^s, and $\bar{\boldsymbol{\beta}}_I = \begin{bmatrix} \bar{b}_{Ix} & \bar{b}_{Iy} & \bar{b}_{Iz} \end{bmatrix}^T$ the smoothed gradient matrix expressed as $\bar{b}_{Ih} = \frac{1}{V_k^s}\sum_{p=1}^{N_\Gamma^s} A_p n_h^p(\mathbf{x}^G) N_I^p(\mathbf{x}^G)$, with N_Γ^s being the total number of boundary surface segments, A_p the area of the p-th boundary surface segment $\Gamma_{k,p}^s$ of the smoothing domain Ω_k^s, and $\mathbf{x}_q^G$ the Gaussian point of the boundary segment $\Gamma_{k,p}^s$, whose unit outward normal vector component is n_h^p, respectively. Note that the integration can be evaluated in the physical coordinate system without a mapping, so that the S-FEM is insensitive with respect to mesh distortion and it allows even concave elements. A typical example of a polyhedral S-FEM based VNE is shown in Fig.1, wherein the element has the eight faces with the individual face centres denoted by green diamonds and the thirteen nodes by blue circles. Here 0_{CPE} and 0_{CPF} indicate the element centre and the centre of an element face, respectively. The blue tetrahedron shows a typical smoothing subdomain or cell for integration when the CS-FEM is adopted. To meet the partition of unity, the shape functions at the central point 0_{CPE} of the element and at the face center 0_{CPF} should be given as $N_I(\mathbf{x}_{0_{CPE}}) = 1/nne$ and $N_I(\mathbf{x}_{0_{CPF}}) = 1/nnf$, where *nne* and *nnf* are the number of the nodes on the polyhedral element and on the element face under consideration, respectively. Furthermore, $N_I(\mathbf{x}_J) = \delta_{IJ}$ for the nodal values of the shape functions, and $N_I(\mathbf{x}_{0_{MP}}) = 1/2$ for the mid points on the element edges. This information on the shape functions is sufficient for the computation of the smoothed displacement gradient or smoothed strain, and so for the computation of the stiffness matrix. Therefore, no explicit expressions for the shape functions are required.

Numerical Examples

In order to demonstrate the accuracy and the convergence behaviour of the proposed scheme, several numerical examples are presented. For the benchmark problem, a three-dimensional cantilever beam subjected to an end-shear load is chosen. The convergence rates of the FEM solutions in terms of the relative errors in the energy norm are given in Fig. 2. This shows that the NS-FEM and the ES-FEM with hexagonal-prism element model give better accuracy with the higher convergence rate than the conventional eight-node hexahedral element.

The proposed SFEM-based VNEs are useful to construct matching meshes and offer good adaptability to complex geometry in practical applications as they offer arbitrary element shapes with arbitrary locations of the element nodes. Moreover, they are extremely advantageous in non-matching problems. As a three-dimensional example, the two-dimensional node-to-node contact scheme of ref. [4] is extended to the three-dimensional node-to-node scheme, which leads to dramatically improved solutions in accuracy and stability than the conventional contact scheme. Note that the polyhedral VNEs are used in this contact scheme to transform non-matching meshes into matching meshes on the contact interface.

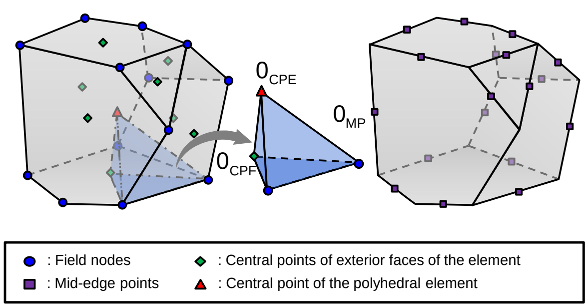

Figure 1. A polyhedral element with an arbitrary number of nodes or faces.

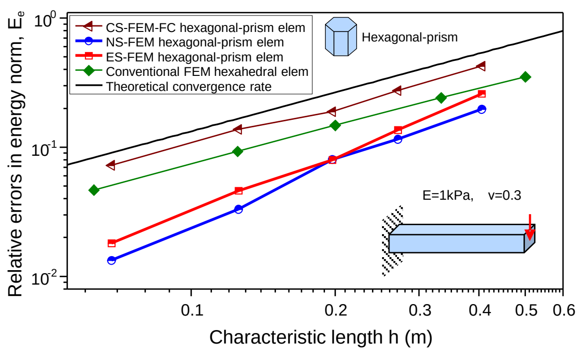

Figure 2. The convergence rates of the polyhedral VNEs and the conventional hexahedral element in the energy norm for the beam under shear load.

First, a three-dimensional flat punch problem, a hard block subjected to a finite-strain compression on a soft basement block, is considered. As shown in Fig. 3(a), the present scheme yields better accuracy in terms of the solution convergence for increasing refinement than Abaqus solution. Furthermore, the Abaqus iterative process for solution fails to converge at the refinement level of a total number of elements greater than 28,825. The difficulty of the convergence of the equilibrium iteration is related to the fact that the contact patch test is not completely passed for the Abaqus model because of the nonmatching contact interface. As noticed in Fig. 3(a), the Abaqus solution for the max. possible refinement of 28,825 elements is yet to level off to give the correct load. On the other hand, Fig. 3(a) shows that the solution from the present CS-FEM based polyhedral VNE approach has already leveled off before the refinement level approximately corresponding to the total 10,000 elements. Second, a hard block subjected to a finite-strain compression sliding on a soft slab with friction is considered. The contact pressure at the point on the block front edge from Abaqus shows severe oscillation with large amplitude, as shown in Fig. 3(b). In contrast, the pressure from the present CS-FEM shows much smaller oscillation.

Note that the preceding approach is very effective also for FSI (Fluid-Solid Interaction) problems, wherein the no-slip condition on the interface between the fluid domain and the solid domain is of critical importance. The use of the S-FEM based VNEs makes it possible to retain the matching mesh throughout the solution by simply adjusting the locations of the nodes on the interface. Thus it becomes possible to impose the no-slip condition precisely. The computation for some three-dimensional FSI problems are currently ongoing, yet to be completed, and will be presented at the conference.

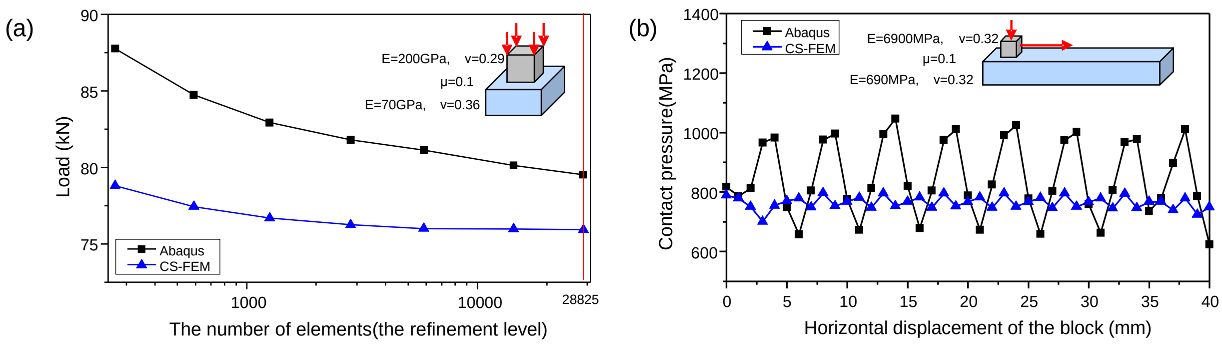

Figure. 3. The results for contact problems using polyhedral VNEs (Neo-Hookean mat'l: E and ν denote the mat'l constants corresponding to Young's modulus and Poisson ratio, and μ the friction coefficient between the two blocks): (a) convergence behavior of the reaction force on the block for the flat punch problem for increasing refinement, (b) the contact pressure of the hard block subjected to compression sliding versus the horizontal displacement.

Conclusions

Through several numerical examples, we have shown that the S-FEM based polyhedral VNEs show an excellent accuracy compared with the conventional FEM, and particularly they are extremely useful in dealing with the nonmatching meshes encountered in contact mechanics and fluid-solid interaction problems.

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