Multiscale constitutive models for particle composites as 'non–simple' continua

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Abstract

The mechanical behaviour of complex materials, characterized at finer scales by the presence of heterogeneities of significant size and texture, strongly depends on their microstructural features. By lacking in material internal scale parameters, the classical continuum does not always seem appropriate to describe the macroscopic behaviour of such materials, taking into account the size, the orientation and the disposition of the micro heterogeneities. This calls for the need of non–classical continuum descriptions (e.g. [1]) obtained through multiscale approaches aimed at deducing properties and relations by bridging information at proper underlying micro–level via energy equivalence criteria.

Focus will be on physically–based corpuscular–continuous models, as originated by the molecular models developed in the 19^{th} century to give explanations 'per causas' of elasticity [4, 8]). Current researches in solid state physics as well as in mechanics of materials show that energy–equivalent continua obtained by defining direct links with lattice systems are still among the most promising approaches in material science. The aim is here to point out the suitability of adopting discrete–continuous Voigt–like models, based on a generalization of the so-called Cauchy–Born rule used in crystal elasticity and in classical molecular theory of elasticity, in order to identify continua with additional degrees of freedom (micromorphic, configurational, etc.). These continua are essentially non–local models with internal length and dispersive properties [8] which, according to the definition in [2], are called non–simple continua. It will be shown as microstructured continuous formulations can be derived within the general framework of the principle of virtual work which, on the basis of a correspondence map relating the finite number of degrees of freedom of discrete models to the continuum kinematical fields, provides a guidance on the choice of the most appropriate continuum approximation for heterogeneous media, allowing us to point out in particular when the micropolar description is advantageous [5, 6, 7, 11].

Basing on the proved effectiveness of continuum micropolar modelling, in particular, some further developments concerning different homogenization methods based on the solution of boundary value problems (BVPs) will be successively introduced. These BVPs are defined at the micro–level and derive from macrohomogeneity conditions of the Hill–Mandel type generalized in order to take into account of additional descriptors, as the relative rotation and curvature degrees of freedom and their dynamic counterparts. These approaches show to be particularly suitable to deal with composite materials characterized by internal structure made of randomly distributed particles of significant size and orientation. It will be shown as a statistically-based multiscale procedure specifically conceived to simulate the actual microstructure a of a random medium at various mesoscale levels allow us to detect the size of representative volume elements, otherwise unknown [3], and to estimate the constitutive moduli of the energy equivalent micropolar continuum [9, 10, 12].

Some applications of the mentioned approaches to fibre reinforced composite materials, ceramic matrix composites and masonry–like material will be reported and discussed.

Keywords: Composite materials, homogenization, micromorphic continua, configurational continua.

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