An Application of the Method of Groebner Bases to a Geometrically Non-linear Free Vibration Analysis of Composite Plates

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Abstract

With the increased use of advanced composite materials throughout industry today, the need for structural members that are comprised of composite layers is becoming more prevalent. In particular, laminated composite plates functioning as structural components with highly nonlinear and coupled nature are finding many applications in modern structures. Therefore, nonlinear modeling and analysis become essential and unavoidable in the design of such plate components, especially under dynamic loading. Like most of the problems in structural mechanics the free vibration of composite plates is often analyzed using linear theories to predict their dynamic behavior. However, if a plate is deflected more than approximately one-half of the plate thickness, especially, if in-plane edge constraints are present, linear assumptions are no longer accurate. Finite element methods have been predominantly used to model large amplitude free vibration problems, but still analytical methods are more preferred, specifically symbolic solutions. All this has provided the motivation for the current study to develop an alternative approach using the method of Groebner bases.

The main application of the method of Groebner bases is to solve a set of nonlinear multivariate polynomial equations. The objective of this study is a further attempt to show its applications in a non-linear free vibration analysis of composite plates and focus on how this technique is employed to solve nonlinear polynomial systems of equations, which usually are solved by numerical methods. In the proposed method, assuming harmonic oscillations the governing integro-(partial) differentiation equation is derived by applying the principle of minimum potential energy. The Ritz method is applied and the displacement shape functions for three different boundary conditions are assumed. Applying variational principle yields a system of non-linear polynomial equations which are in terms of unknown coefficients. Maple 11 is used to determine the reduced Groebner basis for this system of non-linear polynomial equations and obtain symbolic solutions. The results of nonlinear frequency are compared with existing results and have been found to be in close agreement. The effect of aspect ratio, modulus ratio and Poisson's ratio on the non-linear vibration behavior of a fully simply supported plate has been investigated. The proposed method allows us to find relationships, for the purpose of vibration control, among the material properties, vibration amplitude, and natural frequencies, which are impossible to obtain by the numerical methods. The results obtained from the study have shown that the computational method of Groebner bases provides a useful alternative and has a great potential in various engineering applications.

Keywords: Groebner bases, Non-linear vibration, Composite plates