Modelling ideally incompressible hyperelasticity with a new stabilised equal-order mixed formulation: A framework applicable to meshfree, finite element and smoothed finite element methods

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Extended Abstract

Recently meshfree methods and smoothed finite element methods (SFEM) are gaining popularity to model large deformations of hyperelastic materials, due to their relative insensitivity to various conventional mesh-related issues, such as mesh locking and non-physical elemental inversion. While these two methods are more robust than the finite element method (FEM) on this aspect, they present difficulties regarding incompressibility constraints using the mixed formulation, since it is more difficult to satisfy the inf-sup stability condition.

In order to extend SFEM and meshfree methods to model ideally incompressible hyperelasticity, we propose a new mixed formulation. Our scheme approximates both the displacement and pressure fields with the same nodal distribution and basis functions, while the violation of inf-sup condition is compensated for using the Polynomial Pressure Projection (PPP) method [1], which penalises the difference between the unstable pressure field and a more stable lower-order projected pressure field.

Although PPP is proven theoretically and numerically to be unconditionally stable [1], our simulations demonstrated there are minor local instabilities in high stress regions. This is the result of under-integration of rational meshfree basis functions that undermines the stabilisation effect. Therefore, in order to restore stability, we modified the PPP by appending a further stabilisation term to the weak form such that both of the gradients of the original and projected pressures match in a weak sense. The added term can be showed to transform into the conventional pressure Laplacian stabilisation term [2]. Since the method is inherently stable with PPP, the stabilisation parameter associated with the new term is less influential and is set nominally to 1 while still remain effective.

We have validated the proposed scheme against two analytic test cases, including extension/compression of a rubber beam uniaxially and flexion of a rubber beam into an arc [3]. Through these tests, we found that our scheme performs effectively when applied to Galerkin-based FEM, SFEM, and meshfree methods. Our computed solution avoids spurious pressure oscillations and agrees well with the exact solution.

In conclusion, a new and robust stabilised equal-order mixed formulation is proposed and validated to model ideally incompressible hyperelastic materials.

Keywords: Meshfree methods, Hyperelasticity, Incompressibility, Stabilisation method

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