Chemical Reaction, Heat and Mass Transfer on Unsteady MHD Flow along a Vertical Stretching Sheet with Heat Generation/Absorption and Variable Viscosity

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Abstract

The effect of chemical reaction on laminar mixed convection flow and heat and mass transfer along a vertical unsteady stretching sheet is investigated, in the presence of heat generation/absorption with variable viscosity and viscous dissipation. The governing non-linear partial differential equations are reduced to ordinary differential equations using similarity transformation and solved numerically using the fourth order Runge-Kutta method along with shooting technique. The effects of various flow parameters on the velocity, temperature and concentration distributions are analyzed and presented graphically. Skin-friction coefficient, Nusselt number and Sherwood number are derived at the sheet.

Introduction

Processes involving magnetohydrodynamics(MHD) heat and mass transfer flow in the boundary layer, induced by a moving surface in a fluid with chemical reaction occur frequently in nature. It occurs not only due to temperature difference but also due to magnetic field or combination of these. In chemical engineering there are many transport processes that are governed by the joint action of the buoyancy forces from both thermal and mass diffusion in the presence of chemical reaction effects. During a chemical reaction between two species heat is also generated. Diffusion and chemical reactions in an isothermal laminar flow along a soluble flat plate were studied by Fairbanks and Wike [1]. Chakrabarti and Gupta [2] investigated hydromagnetic flow and heat transfer over stretching sheet. Apelblat[3] presented mass transfer with a chemical reaction of first order with effects of axial diffusion.The effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction were studied by Das et al.[4].

Anjalidevi and Kandasamy [5] studied the steady laminar flow along a semi-infinite horizontal plate in the presence of a species concentration and chemical reaction. Fan et al. [6] studied the mixed convective heat and mass transfer over a horizontal moving plate with a chemical-reaction effect. Takhar et al. [7] investigated the flow and mass diffusion of a chemical species with first-order and higher order reactions over a continuously stretching sheet with an applied magnetic field. Muthucumaraswamy [8] studied the effects of a chemical reaction on a moving isothermal vertical infinitely long surface with suction. Anjali Devi and Kandasamy [9] studied effects of chemical reaction, heat and mass transfer on non-linear MHD laminar boundary layer flow over a wedge with suction and injection. Chamkha [10] presented an analytical solutions for heat and mass transfer by laminar flow of a Newtonian, viscous, electrically, conducting and heat generation absorption. The effects of radiation and chemical reactions, in the presence of a transverse magnetic field, on free convective flow and mass transfer of an optically dense viscous, incompressible, and electrically conducting fluid past a vertical isothermal cone surface are investigated by Afify [11]. Kandasamy et al. [12] studied the nonlinear MHD flow with heat and mass transfer characteristics of an incompressible, viscous, electrically conducting and Boussinesq fluid on a vertical stretching surface with chemical reaction and thermal stratification effects.

The combined effects of non-uniform heat source/sink and thermal radiation on heat transfer over an unsteady stretching permeable surface was discussed by Pal [13]. Unsteady mixed convection heat transfer over a vertical stretching surface with variable viscosity and viscous dissipation was studied by Aziz [14]. Radiation and Magnetic field Effects on Unsteady Mixed Convection Flow over a Vertical Stretching/Shrinking surface with suction/injection was discussed by Sandeep et al[15].

The objective of the paper is to investigate the influence of heat and mass and magnetic field on an unsteady flow over a vertical stretching sheet with heat generation/absorption and chemical effects in presence of variable viscosity and viscous dissipation.

Formulation of the problem

An unsteady, two-dimensional, boundary-layer convective flow of an incompressible, viscous and electrically conducting fluid along a vertical stretching sheet embedded in porous media in the presence of heat and mass transfer, chemical reaction is considered. The x-axis is considered along the sheet and y-axis is perpendicular to the sheet. The fluid properties are assumed to be constant except the viscosity, the density term of in buoyancy terms of the momentum equations and the chemical reaction is homogeneous and of first order taking place in the flow. The sheet is stretching in its own plan with velocity

$$u_w=bx/(1-\alpha t) \tag{1}$$

where $b(>0)$ is the stretching parameter and $\alpha(>0)$ is the unsteadiness parameter and both have dimensions of $(\text{time})^{-1}$. The surface temperature T_w and concentration distribution of the sheet C_w, which varies with the distance x along the sheet and time t. The system is influenced by an external transverse magnetic field of strength B defined as

$$B=B_0(1-\alpha t)^{-1/2} \tag{2}$$

The volumetric rate of heat generation/absorption is given as

$$Q=Q_0(1-\alpha t)^{-1} \tag{3}$$

Under above assumptions, the governing equations of continuity, momentum, energy and concentration are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{4}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)+g\beta(T-T_\infty)+g\beta^*(C-C_\infty)-\frac{\sigma B^2}{\rho_\infty}u \tag{5}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho_\infty C_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho_\infty C_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{Q}{\rho_\infty C_p}(T-T_\infty) \tag{6}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-K_1(C-C_\infty) \tag{7}$$

along with the boundary conditions

$$\begin{gathered}\text{at } y=0:\ u=u_w,\ v=0,\ T=T_w,\ C=C_w \text{ and}\\ \text{as } y\rightarrow\infty:\ u\rightarrow 0,\ T\rightarrow T_\infty,\ C\rightarrow C_\infty\end{gathered} \tag{8}$$

where u and v are the velocity components along the x and y directions respectively, ρ_∞ is the density of the fluid, g is the gravitational acceleration, β is the thermal expansion coefficient, β^* is the concentration expansion coefficient, σ is the electrical conductivity, T is fluid temperature inside the thermal boundary layer, C is the species concentration in boundary layer, T_∞ is the temperature far away from the sheet, C_∞ is the species concentration far away from the sheet. C_p is the specific heat at constant pressure, k is the thermal conductivity, D is the mass diffusion coefficient. Variation of the viscosity with temperature are assumed to be of the form[16]

$$1/\mu = [1+\tau(T-T_\infty)]/\mu_\infty = a(T-T_r), \tag{9}$$

$$\text{where } a=\tau/\mu_\infty \text{ and } T_r=T_\infty-1/\tau \tag{10}$$

are constants and their values depend on reference state and the thermal property of the fluid τ. Also $T_w=T_\infty+[bx^2/2\nu_\infty](1-\alpha t)^{-2}$ and $C_w=C_\infty+[bx^2/2\nu_\infty](1-\alpha t)^{-2}$, where ν_∞ is the kinematic viscosity of the fluid.

Introducing the similarity variable η and the dimensionless variables f, θ and ϕ as:

$$\eta=(b/\nu_\infty)^{1/2}(1-\alpha t)^{-1/2}y \tag{11}$$

$$\psi=[\nu_\infty b/(1-\alpha t)]^{1/2}x.f(\eta) \tag{12}$$

$$T=T_\infty+[bx^2/2\nu_\infty](1-\alpha t)^{-2}\theta(\eta) \tag{13}$$

$$C=C_\infty+[bx^2/2\nu_\infty](1-\alpha t)^{-2}\phi(\eta) \tag{14}$$

where $\psi(x,y,t)$ is the stream function satisfying the continuity equation (4) with $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$. The components of velocity can be readily expressed as:

$$u=u_w f'(\eta),\ v=-[\nu_\infty b/(1-\alpha t)]^{1/2}f(\eta) \tag{15}$$

Making use of Eqs. (11)-(14), Eqs. (5)-(7) reduce to

$$f'''=\frac{\theta_v-\theta}{\theta_v}[A(f'+0.5\eta f'')+f'^2-ff''-\lambda(\theta+N\emptyset)+Mf']-\frac{\theta' f''}{\theta_v-\theta} \tag{16}$$

$$\theta''=Pr\left[A(2\theta'+0.5\eta\theta')-f\theta'+2f'\theta-S\theta-\frac{\theta_r}{\theta_v-\theta}Ecf''^2\right] \tag{17}$$

$$\theta''=Sc[A(2\phi+0.5\eta\phi')-f\phi'+2f'\phi+\gamma\phi] \tag{18}$$

The transformed boundary conditions:

$$\text{at } \eta=0:\ f=0,\ f'=1,\ \theta=1,\ \phi=1 \text{ and}$$
$$\text{at } \eta\to\infty:\ f'\to 0,\ \theta\to 0,\ \phi\to 0. \tag{19}$$

where a prime denotes ordinary differentiation with respect to η, $\theta=(T-T_\infty)/(T_w-T_\infty)$ is the non-dimensional temperature, $\theta_v=(T_r-T_\infty)/(T_w-T_\infty)=-1/\tau(T_w-T_\infty)$ is the variable viscosity parameter, $A=\alpha/b$ is the unsteadiness parameter, $Ec=2b\nu_\infty/C_p=u_w^2/C_p(T_w-T_\infty)$ is the Eckert number, $M=2\sigma B_0^2(1-\alpha t)/\rho_\infty b$ *is the Magnetic number,* $N=\beta^*(C_w-C_\infty)/\beta(T_w-T_\infty)$ *is the Buoyancy ration parameter*, $Pr=\nu_\infty\rho_\infty C_p/k$ is the Prandtl number, $S=Q(1-\alpha t)/a\rho C_p$ is the heat generation/absorption Parameter and $\lambda=g\beta x/2b\nu_\infty=Gr_x/Re_x^2$ is the mixed convection parameter with $Gr_x=g\beta(T_w-T_\infty)x^3/\nu_\infty^2$ is the Grashof number. The case in which $\lambda=0$ corresponds to the forced convection regime while that in which λ is large corresponds to thefree convection regime.

For practical applications, the physical quantities of major interest are the local friction coefficient C_{fx}

$$C_{fx}=2\mu(\partial u/\partial y)_{y=0}/\rho_\infty u_w^2=(2\theta_v/\theta_v-1)Re_x^{-1/2}f''(0) \tag{20}$$

the local Nusselt number Nu_x

$$Nu_x=-x(\partial T/\partial y)_{y=0}=-Re_x^{3/2}\theta'(0)/[2(1-\alpha t)] \tag{21}$$

and local Sherwood number Sh_x

$$Sh_x=-x(\partial C/\partial y)_{y=0} = -Re_x^{3/2}\phi'(0)/[2(1-\alpha t)] \tag{22}$$

where $Re_x = u_w x/\nu_\infty$ is the local Reynolds number based on the sheet velocity u_w.

Results and discussion

The non-linear coupled differential Eqs. (16)-(18) with boundary condition (19) and constitutes a boundary value problem has been solved numerically by fourth order Runge-Kutta Shooting method for different values of the parameters. Effect due to magnetic field and chemical reaction at the wall of the cone over the velocity, temperature and concentration are shown through figures 1-3. Fig. 1 depicts the dimensionless velocity profiles for different values of magnetic field and chemical reaction parameters. It observes that the velocity component of the fluid along the surface of the sheet increase with decrease of the strength of the magnetic field, on the contrary, fig. 2 and 3 shows the dimensionless temperature and concentration of the fluid increase with increase of the strength of the magnetic field. On the other hand the dimensionless velocity and temperature of the fluid reduce with an increase of chemical reaction parameter while the dimensionless concentration has the opposite behavior.

Figures 4-6 present the effects of the unsteadiness parameter A on the velocity, temperature and concentration profiles, respectively. From these figures it observed that increasing value of A results in decreasing the velocity, temperature and concentration keeping other parameters fixed. Figures 7-9 illustrate the influence of the chemical reaction parameter γ and the Schmidt number Sc on the velocity, temperature and concentration profiles in the boundary layer, respectively. Increasing the chemical reaction parameter produces a decrease in the species concentration. In turn, this causes the concentration buoyancy effects to decrease as γ increases. Consequently, less flow is induced along the sheet resulting in decreases in the fluid velocity in the boundary layer. On the other hand, the fluid temperature increases as γ increases. In addition, the concentration boundary layer thickness decreases as γ increases. Moreover, the Schmidt number is an important parameter in heat and mass transfer processes as it characterizes the ratio of thicknesses of the viscous and concentration boundary layers. Its effect on the species concentration has similarities to the Prandtl number effect on the temperature. That is, increases in the values of Sc cause the species concentration and its boundary layer thickness to decrease resulting in less induced flow and higher fluid temperatures. This is depicted in the decreases in the velocity and species concentration and increases in the fluid temperature as Sc increases. These behaviors are clearly evident in Figures 7-9. The influence of heat generation/absorption over velocity, temperature and concentration are elucidated with the help of figures 10-12. It is clear that the velocity of the fluid increases with increase of heat generation parameter S but the temperature and concentration of the fluid increases with increase of S. On the other hand the velocity, temperature and concentration of the fluid decrease with the increasing values chemical reaction parameter γ.

Conclusions

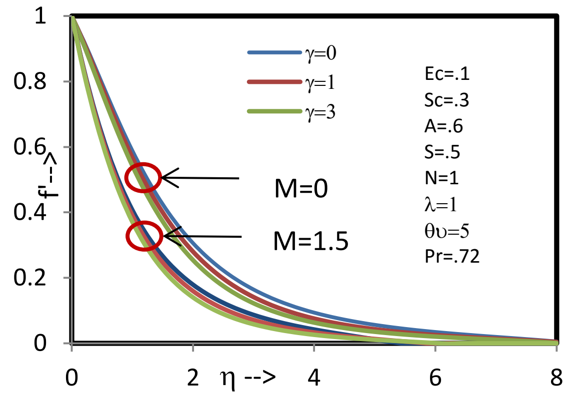
We conclude the following from above results and discussion:

1.The influence of chemical reaction, the fluid flow along the sheet accelerate with increase of chemical reaction parameter, on the other hand, temperature of the fluid increases with increase of chemical reaction parameter but concentration of the fluid reduces with it.

2. For all values of unsteadiness parameter, increasing values of the chemical reaction parameter the boundary layer decreases on the surface of the sheet.

3. The increases in the values of *Sc* cause the species concentration and its boundary layer thickness to decrease resulting in less induced flow and higher fluid temperatures. This is depicted in the decreases in the velocity and species concentration and increases in the fluid temperature as *Sc* increases.

4. Due to heat generation, increases of heat generation parameter accelerate the fluid motion and decelerate the temperature and concentration of the fluid along the sheet.



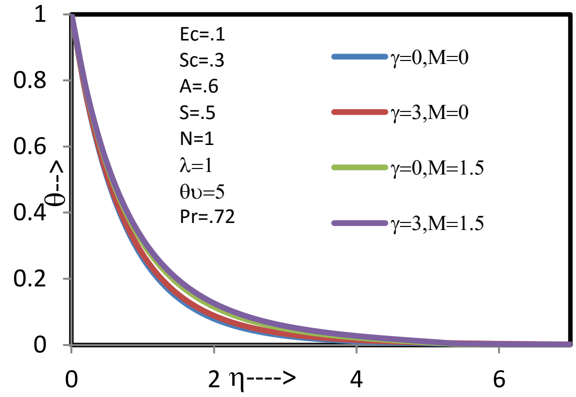
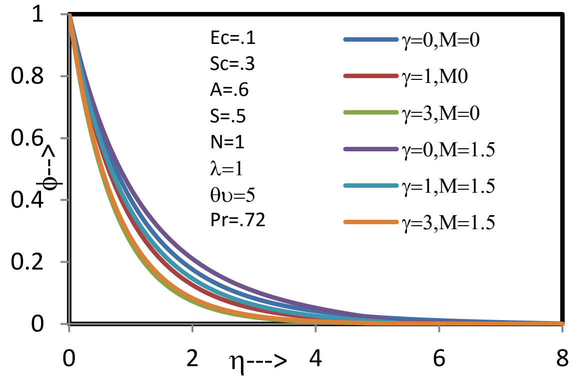


Fig.1 Velocity profiles for various values of chemical reaction γ and magnetic parameter M.

Fig.2 Temperature profiles for various values of chemical reaction γ and magnetic parameter M.



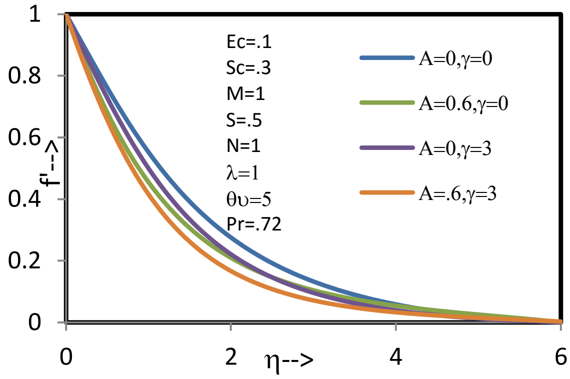
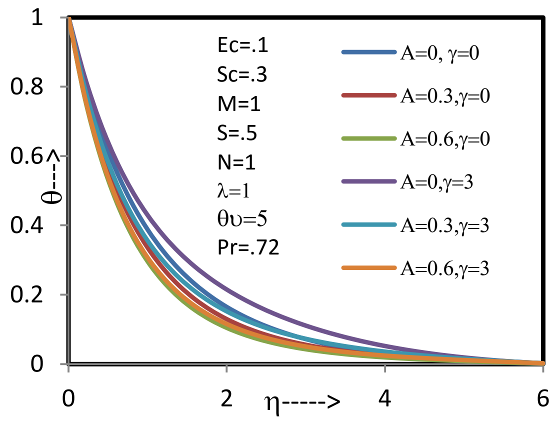


Fig. 3 Concentration profiles for various values of chemical reaction γ and magnetic parameter M.

Fig.4 Velocity profiles for various values of chemical reaction γ and unsteady parameter *A*



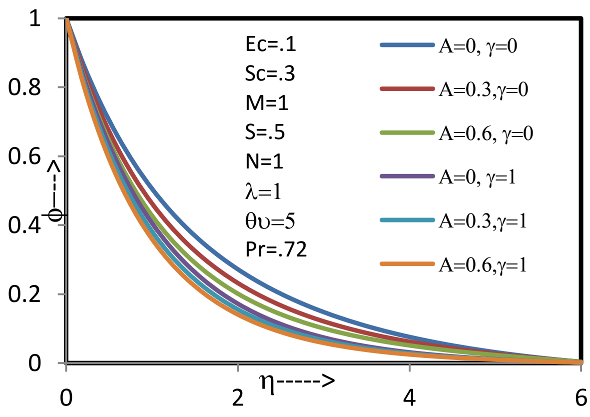
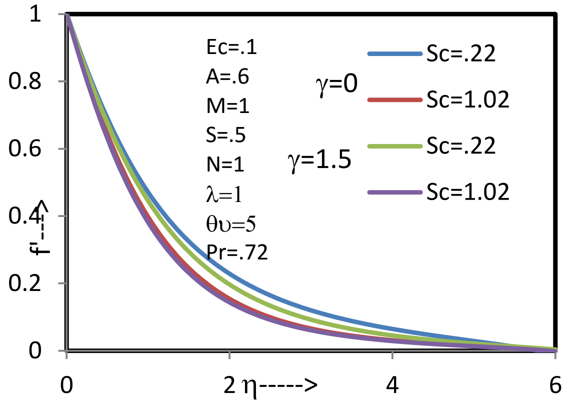


Fig. 5 Temperature profiles for various values of chemical reaction γ and unsteady parameter *A*

Fig. 6 Concentration profiles for various values of chemical reaction γ and unsteady parameter *A*.



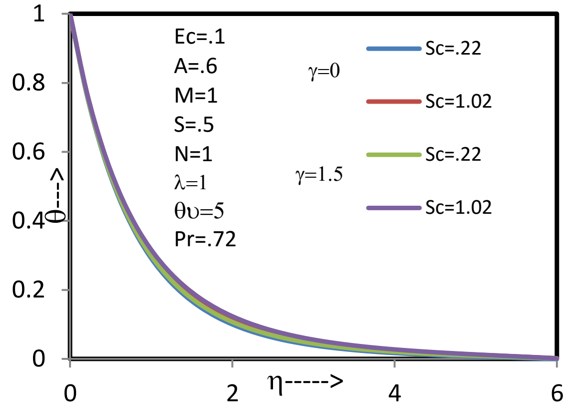
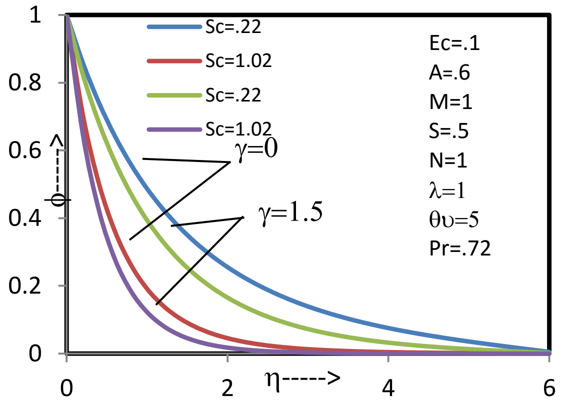


Fig.7 Velocity profiles for various values of chemical reaction γ and Schmidt number Sc.

Fig.8 Temperature profiles for various values of chemical reaction γ and Schmidt number Sc.



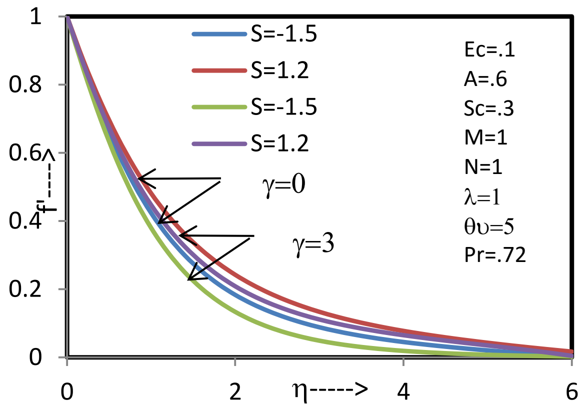
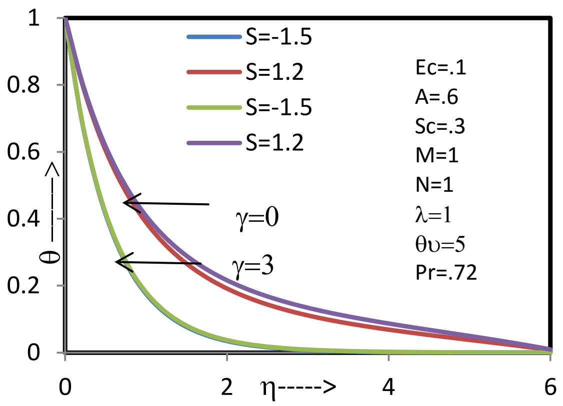


Fig. 9 Concentration profiles for various values of chemical reaction γ and Schmidt number Sc.

Fig.10 Velocity profiles for various values of chemical reaction γ and heat generation S.



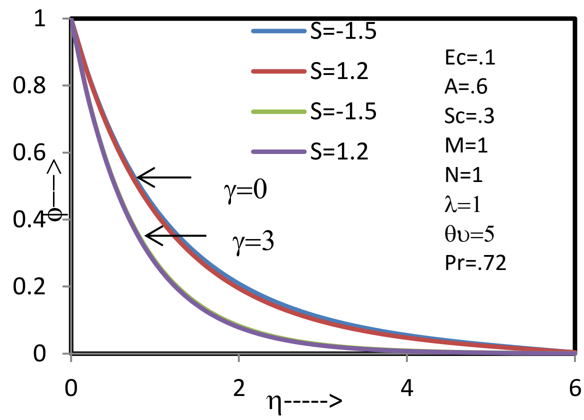


Fig.11 Temperature profiles for various values of chemical reaction γ and heat generation S.

Fig. 12 Concentration profiles for various values of chemical reaction γ and heat generation S.

References

[1] Fairbanks, D.F. and Wike, C.R.(1950) Diffusion and chemical re-action in an isothermal laminar flow along a soluble flat plate, Ind. Eng. Chem. Res., 42, 471- 475.

[2] Chakrabarti, A. and Gupta A. S.(1979) Hydromagnetic flow and heat transfer over stretching sheet, Quarterly Journal of Mechanics and Applied Mathematics, 37, 73-78.

[3] Apelblat A.(1982) Mass transfer with a chemical reaction of the first order: effects of axial diffusion, *The Chemical Engineering Journal*, 23, 193-203.

[4] Das, U.N., Deka, R.A. and Soundalgekar, V.M.(1994) Effect of Mass Transfer on Flow Past an Impulsively Started Infinite Vertical Plate with Constant Heat Flux and Chemical Reaction, Forschung im Ingenieurwesen, 60, 284-287.

[5] Anjalidevi, S. P. and Kandasamy, R. (1999) The steady laminar flow along a semi-infinite horizontal plate in the presence of a species concentration and chemical reaction. *Heat Mass Transfer*, 35 465-467.

[6] J.R. Fan, J.M. Shi, and X.Z. Xu, (1998) The mixed convective heat and mass transfer over a horizontal moving plate with a chemical-reaction effect. *Acta Mech.*, 126, 59-69.

[7] Takhar, H.S., Chamkha, A.J. and Nath, G.(2000) Flow and mass transfer on a stretching sheet with a magnetic field and chemically reactive species, Int. J. Eng. Sci., 38, 1303-1314.

[8] Muthucumaraswamy, R. (2002) Effect of a Chemical Reaction on a Moving Isothermal Vertical Surface with Suction", Acta Mechanica, 155, 65-70.

[9] Anjali Devi S.P. and Kandasamy, R. (2002) Effects of Chemical Reaction, Heat and Mass Transfer on Non-Linear MHD Laminar Boundary Layer Flow over a Wedge with Suction and Injection, Int. Comm. Heat Mass Transfer, 29, 707-716.

[10] Chamkha, A.(2003) MHD Flow of Uniformly Stretched Vertical Permeable Surface in the Presence of Heat Generation/Absorption and a Chemical Reaction, Int. Comm. Heat Mass Transfer, 30, 413-422.

[11] Afify, A.A.(2004) The effect of radiation on free convective flow and mass transfer past a vertical isothermal cone surface with chemical reaction in the presence of a transverse magnetic field, Can. J. Phys., 82, 447-458.

[12] Kandasamy, R., Periasamy, K. and Sivagnana Prabhu, K.K.(2005) Chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with heat source and thermal stratification effects, Int. J. of Heat and Mass Transfer, 48, 4557-4561.

[13] Pal D.(2011) Combined effects of non-uniform heat source/sink and thermal radiation on heat transfer over an unsteady stretching permeable surface, *J. Communications in Nonlinear Science and Numerical Simulation*. 16 (4), 1890-1904.

[14] Aziz, M.A.E. (2014). Unsteady mixed convection heat transfer along a vertical stretching surface with variable viscosity and viscous dissipation, J. of the Egyptian Mathematical Society, 22, 529–537.

[15] Sandeep N, Sulochana C., Sugunamma V.(2015) Radiation and magnetic field effects on unsteady mixed convection flow over a vertical stretching/shrinking surface with suction/injection, *Industrial Engineering Letters*, 5(5), 127-136.

[16] Lai, F.C. and Kulacki, F.A. (1990) The effect of variable viscosity on convective heat transfer along a vertical surface in a saturated porous medium, Int. J. Heat Mass Transfer 33(5), 1028-1031.