Modeling Complex Dynamical Systems in MF Range Combining FEM and Energy Methods

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Abstract

Complex dynamical systems such as car body, aircraft fuselage or train coach are conveniently modeled with Finite Element Method (FEM) in the Low Frequency range (LF). Increasing the frequency range to Mid-Frequencies (MF), typically up to 1000-2000 Hz, requires larger and larger FEM mesh. Presently, MF fluid/structure interaction problems on large structures cannot be solved in decent time at engineering level. Reduction of model size is required especially under random distributed loads. Energy methods like Statistical Energy Analysis (SEA) provide a theoretical framework for building small models based on power-balanced- equations they can be run in High Frequency range (HF). Nevertheless, SEA parameters are derived from analytical solutions of differential operators and submitted to many assumptions and simplifications. They cannot provide robust enough prediction in MF range due to inherent complexity of industrial systems.

To improve predictability of energy models, the relevant parameters are then identified by inverse method from the “statistical” dynamic information contained in side FEM model. The FEM-derived SEA models are called Virtual SEA models (VSEA). They use the same parameters than the classical “analytical” SEA models. VSEA parameters can then be directly compared to their analytical counterparts. VSEA models may be understood as compressed FEM models in which the narrow-band frequency and spaced-varying FEM dynamic is replaced by band-integrated frequency and spaced averaged dynamic applied to a partition of FEM domain into subsystems. This compression leads to very small models while minimizing the information depredation. For example car body-in-white dynamic described by 6 million DoF’s in FEM is encapsulated as a real-valued 50x50 matrix relating injected power from impressed forces to energy in each of the subsystems. Problems involving random loads can then be solved by using VSEA models rather than original FEM’s. VSEA models can also be complemented by analytical other subsystems such as fluid cavities to solve full vibroacoustic response involving airborne and structure-borne propagation paths. Outputs from VSEA models are also more easily interpreted and provide description of propagation paths in the system.

Keywords: Statistical Energy Analysis, SEA, Virtual SEA, VSEA, Computational Dynamic, Propagation path.

Introduction

SEA [1] has been and is still a very popular method in vibroacoustic engineering to predict random vibrations and fluid –structure interaction over a broadband frequency range. SEA describes the interaction of subdomains of a given dynamical systems in term of energy through a set of power-balanced equations. To build a valid SEA representation of the actual vibrational state, the system needs to be partitioned into subsystems and coupling coefficients calculated with appropriate physical laws. Some restrictive assumptions such as “weak” subsystem coupling must also be fulfilled. The three latter tasks have been for a long time the drawback of the SEA method due to lack of guidance in constructing models of complex systems. The use of analytical dynamical operators for computing SEA parameters was also limiting engineers in their ability to handle the structural complexity.

Nevertheless over years, despite all these obstacles, SEA method has been successful in predicting responses of a large class of industrial systems but generally with lack of control over frequency-band modeling limit and variance. From early 2000$^{th's}$ SEA modeling capabilities have been leveraged up over Mid-Frequency (MF) by relying on Finite Element Method (FEM) to provide more robust SEA representation using FEM derived-parameters. This technology called Virtual SEA (VSEA) has enlightened the general dynamical behavior of complex systems, leading as side effects to improvement of equivalent analytical modeling rules always required for extending VSEA results above the frequency limit of the FEM mesh. There are theoretical connections with the parallel development of stochastic FEM modeling over the MF range [2][17].

The SEA Power Equilibrium

A given dynamical system is partitioned into two subsystems for easier presentation. Between the resulting two subsystems, a set power-balanced equations Eq. (1) traduces the conservation of energy. Power flowing into a subsystem, from either a local source or arising from another coupled subsystem, is, for a fraction, dissipated into the subsystem and for another fraction, sent back to the coupled subsystem.

$$\begin{cases} \pi_1 / \omega_c = \eta_1 E_1 + \eta_{12} E_1 - \eta_{21} E_2 \\ \pi_2 / \omega_c = \eta_2 E_2 + \eta_{21} E_2 - \eta_{12} E_1 \end{cases} \tag{1}$$

$\eta_1 E_1$, $\eta_2 E_2$ are the vibrational energies dissipated by subsystem 1 and 2. $\pi_{12} / \omega_c = \eta_{12} E_1 - \eta_{21} E_2$ is the net power flow between subsystems expressed as the difference of radiated energies from 1 to 2 and 2 to 1 and $\pi_{21} = -\pi_{12}$.

π_1 and π_2 terms represent the injected power by external random loads in resp. subsystem 1 and 2 over a frequency band of width B centered around radian frequency ω_c.

η_1 and η_2 are the mean modal Damping Loss Factors of the subsystems (or DLF) and η_{12}, η_{21} the Coupling Loss Factors between related subsystems (CLF).

Under external random impressed loads, the subsystem vibrational energy is related to the frequency and spaced-averaged mean squared velocity $< \bar{v}^2 >$ times the subsystem mass:

$$E = m < \bar{v}^2 > \tag{2}$$

Injected power due to random loads is independent of the modal subsystem response. Therefore, knowing DLF and CLF, we can build a loss matrix relating injected power vector to energy vector and calculate the energies in function of injected power in the band *B*:

$$\begin{pmatrix} E_1 \\ E_2 \end{pmatrix} = \begin{pmatrix} \eta_1 + \eta_{12} & -\eta_{21} \\ -\eta_{12} & \eta_2 + \eta_{21} \end{pmatrix}^{-1} \begin{pmatrix} \pi_1 \\ \pi_2 \end{pmatrix} / \omega_c \tag{3}$$

For Eq. (3) being a valid representation of the actual energy exchange between the two subsystems, impressed powers π_1 and π_2 need to be uncorrelated. For describing the energy exchange between two continuous bounded subsystems (two plates or a plate and an acoustic cavity), the concept of modal energy is introduced. In a band *B*, the continuous subsystem is statistically described by a set of modal oscillators carrying modal energy. If *N* oscillators are resonating in *B*, the statistic modal energy is equal to $\varepsilon = E / N$ where *E* is the sum of the *N* modal energies. Considering two set of modal oscillators resonating in *B* with respectively N_1 and N_2 number of resonance frequencies in *B*, SEA states that the net power flow between the set is expressed by Eq. (4).

$$\pi_{12} = N_1 N_2 \beta\left(\varepsilon_1 - \varepsilon_2\right) \tag{4}$$

It leads the symmetrical SEA power-balanced equations given by Eq. (5).

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix} = \begin{pmatrix} E_1 / N_1 \\ E_2 / N_2 \end{pmatrix} = \left[\begin{pmatrix} (\eta_1 + \eta_{12}) N_1 & -\eta_{21} N_2 \\ -\eta_{12} N_1 & (\eta_2 + \eta_{21}) N_2 \end{pmatrix}\right]^{-1} \begin{pmatrix} \pi_1 \\ \pi_2 \end{pmatrix} / \omega_c \tag{5}$$

Off-diagonal terms are equal due to reciprocity relationship, $N_1\eta_{12} = N_2\eta_{21}$, making the loss matrix symmetrical as the modal coupling coefficient β is always symmetrical due to linearity of dynamical operators.

The modal formulation of SEA provides an easier interpretation of the "weakly" coupled assumption. Under this assumption, the subsystem modes can be considered as an acceptable orthonormal basis for projecting its responses and the related total energy is then found much closer to the discrete sum of modal energies. If subsystems 1 and 2 are strongly coupled, their modes are hybridized and undistinguishable with non-null cross-correlated energy E_{12}.

The further required assumption of "weak" coupling between subsystems has been discussed for a long time among SEA community and it is only recently that the role of this requirement has been clarified: Eq. (4) is always valid as soon as we restrict the calculation of energy to resonant modes. But on one hand, in case of more than two coupled subsystems, CLF law coupling related oscillators is different from the weak coupling case and on the other hand, all subsystems are found cross-coupled together, independently they are connected or not on a physical boundary.

The degree of physical coupling between thin shells, parts of a complex system, is generally decreasing with frequency as any small discontinuity of mass or stiffness in the system will generate growing reflection coefficient when frequency increases (i.e. wavelength decreases). It leads to progressive confinement of energy in localized subdomains of the dynamical system. Confined energy is thus stored in local modes of the subsystems creating energy gaps between subdomains and SEA representation given by Eq. (4) and Eq. (5) will start to work and calculated CLF can be restricted to near-by coupled subsystems. As a consequence, there is always some cut-off frequency under which SEA scheme will be found defective.

The EDM Power Equilibrium

When subsystems are strongly coupled, power flow is no more discontinuous as in Eq. (4) as the distribution of energy density within the union of subsystems is continuous function of space. The Energy Diffusion Method (EDM) demonstrates [4][5][6] that in a continuous uniform system of extension Ω the conservation of energy between subdomains $\delta\Omega$ is given by the following energy-based equation:

$$-\frac{c_g^2}{\eta\omega}\Delta e + \eta\omega e = \pi_{inj} \tag{6}$$

where c_g is the group velocity of underlying propagating waves and e the energy density in the medium. η is the mean DLF in the medium in which is assumed perfect diffusion of the vibrational field. The local intensity is found proportional to the gradient of energy density:

$$\vec{I} = -\frac{c_g^2}{\eta\omega}\vec{\nabla}e \qquad (7)$$

The coefficient $\frac{c_g^2}{\eta\omega}$ plays then the same role than the coefficient of thermal transfer in heat exchange problems.

SEA and EDM Power Flows

A large fluid cavity is now considered containing many acoustic modes and its volume is split into smaller cavities. SEA is assuming weak coupling between subsystems. Therefore such an SEA model cannot represent actual distribution of energy in the volume as obviously the coupling is expected to be very strong between two neighboring sub-volumes as their boundaries are not at all reflective. To illustrate it, an acoustic volume (12m x 1m x 1m) is split into two different partitions. For comparison of calculated local energies, two SEA models are built from the partitions with sub-volumes cross-coupled by the “regular” SEA CLF.

Partition C6 is made of 6 sub-volumes (2m x 1m x 1m each) while partition C12 is made of 12 sub-volumes (1m x 1m x 1m) as sketched in Figure 1.

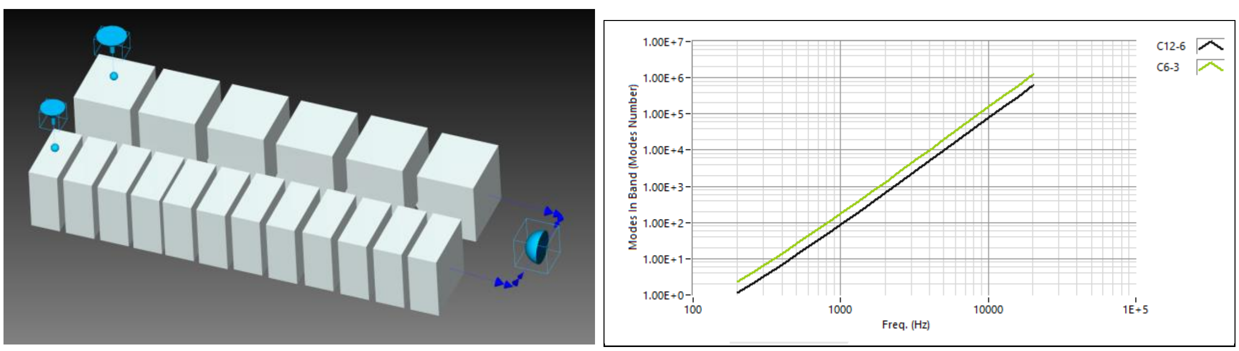

Figure 1. Two SEA partitions of the same acoustic volumes and number of modes per 1/3rd octave band in the two related elementary sub-volumes

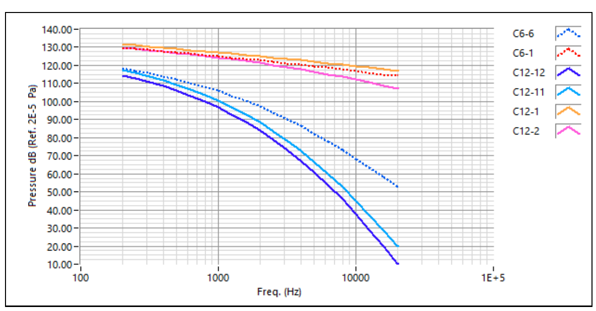

Figure 2. Pressure levels in first and last acoustic sub-volumes in C6 and C12 partitions for unit power injected in resp. C6-1 and C12-1

Elementary sub-volumes are all including at least one acoustic resonance from 200 Hz. When applying a unit source power in the first left volumes, we expect to find similar pressure level at right ends of both partitions. Sub-volumes are numbered C6-1, C6-2… for partition C6 and C12-1, C12-2… for partition C12. Figure 2 shows pressure levels calculated in both models in first and last sub-volumes. For C12 volumes, the two first and the two last are graphed as they are half-sized compared to C6 corresponding volumes. By doubling the number of sub-volumes to mesh the total volume, predicted pressure in C12 drops down from about 25 to 30 dB at 20 kHz and of about 10 dB around 2 kHz. The predicted pressure level is found dependent on mesh size (i.e. to the number of subsystems used to describe the volume).

Same exercise may be done in any SEA software method and will lead to similar result if CLF are computed from wave transmission method.

In this particular example of serial energy transfer within an arbitrary partition of an acoustic volume, weak coupling assumption is not verified. Therefore the volume meshing is not consistent.

EDM provides a more representative model to describe the actual energy transfer based on a different power flow formulation. For computing CLF, classical SEA method, as originally proposed [1] - and still applied in the community of SEA users - relies on wave theory and weak coupling. At subsystem boundary, the power flow from an emitter subsystem to a receiver one, is then given by:

$$\Pi_{1\to 2} = \eta_{12}\omega E_1 = \langle \tau \rangle_\theta I_{inc}\Sigma \tag{8}$$

Where $\langle \tau \rangle_\theta$ is the mean random-incidence wave transmission coefficient, I_{inc} is the incident intensity propagating from 1 to 2 and Σ the junction size (area for 3D acoustic volumes). Because $I_{inc} = \frac{c}{4}e$ in the diffuse acoustic field, e being the energy density, Wave Transmission (WT) CLF is found equal to:

$$\eta_{12} = \frac{\langle \tau \rangle_\theta c\Sigma}{4\omega\Omega} \tag{9}$$

For two identical coupled sub-volumes, the net power flow is finally expressed in function of WT CLF as:

$$\pi_{12} = \frac{c\Sigma}{4\Omega}\left(E_1 - E_2\right) = -\frac{c}{4}\cdot\frac{E_2 - E_1}{L} \tag{10}$$

where is L=Ω/Σ is the characteristic length of the sub-volumes and $\langle \tau \rangle_\theta$=1 as no refection occurs at sub-volume interface.

Comparing Eq. (10) and Eq. (7), both net power flow expressions are found proportional to the gradient of energy (or energy density with appropriate scaling). But the coefficients of vibrational transfer that relates the gradient of energy to power are following different laws vs. frequency.

Therefore, in any complex dynamical system, the energy distribution over the domain is expected to be continuous over sub-domains with low reflective boundaries between them and submitted to step when crossing a reflective boundary. Because the reflectivity of a boundary is frequency-dependent when speed of sound slightly varies between two coupled sub-domains, we have a better understanding of Figure 2 result: the discretization of an acoustic volume into sub-volumes coupled by WT CLF implicitly creates artificial loss of energy at each boundary and leads to a larger energy drop in the last cavity. This effect is a consequence of applying WT CLF between strongly-coupled regions. Fluid-structure interaction problems are less entailed by this problem as fluid and structure, at least for air, are always weakly coupled.

SEA method based on WT-CLF is not representative of actual dynamical behavior in analyzing structure borne noise propagation over the MF range when speed of sound between the various parts is not very different as in the case of car body-and-white where the skin is everywhere between 0.7 to 1 mm thick with low reflectivity from boundaries.

Over MF range, for improving robustness of SEA modeling, WT CLF should be only applied for coupling regions separated by steep energy gradient while continuous regions with smooth gradient should be either considered as a single SEA subsystem or split into smaller scale subsystems coupled by EDM modified CLF. Figure 3 shows the prediction with an SEA model with a topology similar to Figure 1 but built following previous recommendations (i.e. with different CLF expression). Sub-volumes in partitions C6 and C12 are cross-coupled by discretized EDM CLF in SEA+ software [15] and again we plot pressure levels in first and last acoustic volumes in Figure 3. In that case, the mesh dependence has nearly been cleared. Pressure in the two last C12 sub-volumes are now close to C6 related sub-volume as expected in the actual physics.

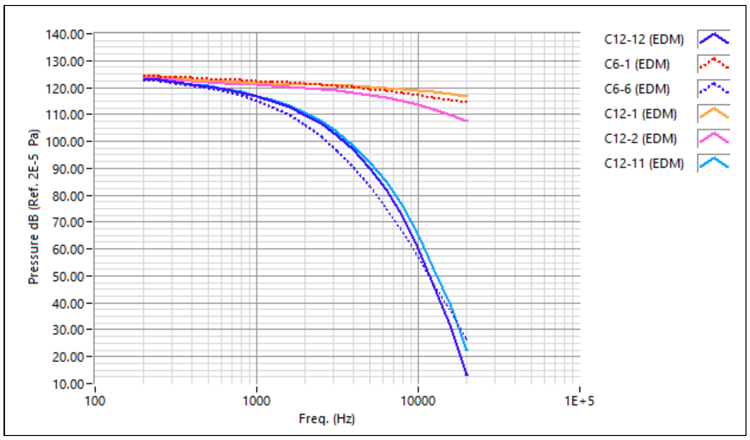

Figure 3. Pressure levels in first and last acoustic sub-volumes in C6 and C12 partitions for unit power injected in resp. C6-1 and C12-1 applying EDM CLF between sub-volumes

Virtual SEA for SEA Prediction over MF Range in Complex Structures

Creation of a representative SEA model requires discriminating subdomains separated by expected energy gap, in function of the vibrational frequency reflectivity of boundaries as previously discussed. Partition into subsystems is then a key feature in designing a “physical” SEA model but is an unknown of the vibration problem.

For complex dynamical system such car body, aircraft, spacecraft or any compact structural component such an electronic equipment, EDM theory cannot be directly applied as constant speed of sound is required among regions of smooth energy distribution. Finding potential energy gaps between various zones is also not always intuitive. Based on expertise developed in measuring SEA CLF on built-up structures [8][9][10], VSEA method [11][12][13] was introduced for characterizing SEA CLF and weakly coupled regions of car chassis, further extended to spacecraft analysis and full car body or components in operating conditions.

For non-homogeneous systems with complex geometry, VSEA relies on FEM global real modes to get a snapshot of the statistical vibrational behavior in the various targeted frequency bands. VSEA is derived from experimental SEA analysis or ESEA [7] and identifies SEA parameters of the system responses by solving an inverse SEA problem like in ESEA method. Inputs to the inverse problem are the synthesized modal responses at a grid of predefined nodes. VSEA may be viewed as a kind of virtual test where the system dynamics is reduced to responses at a subset of discrete nodes.

Virtual SEA Numerical Process

Real modes are extracted using preferred FEM solver (NX-NASTRAN in next example). Eigenfrequencies and related modal amplitudes, at a set of restitution nodes, are stored and exported to SEA+ VSEA solver. VSEA is synthesizing complex velocity FRF $v_i/f_j = v_{ij}$ at all restitution points M_i in global x, y, z directions due to rain-on-the-roof unitary x, y, z forces applied at each restitution node M_j. Final FE statistical information is reduced to the transfer velocity matrix **V** made of v_{ij} elements

Global DLF for modal synthesis is taken equal to some frequency band-dependent default value for all modes. **V** matrix is compressed into 1/3rd octave band and projected in the direction n_i and n_j of maximal input/output conductances given for at all nodes by Y =Re{Diag(**V**)}n. The final matrix $\bar{\mathbf{V}}^2$ for SEA-parameter identification is then expressed in band-averaged format at center radial frequency ω_c and bandwidth B with elements given by:

$$v_{ij}^2(\omega_c, B) = \frac{1}{B}\int_B v_{ij}^2(\omega)d\omega \tag{11}$$

$\bar{\mathbf{V}}^2$ is finally auto-partitioned by SEA+ peripheral algorithm which groups nodes into a set of weakly coupled subsystems Ω_k leading to SEA rectangular transfer matrix $\langle \bar{\mathbf{V}}^2 \rangle$ of which elements are given by:

$$\langle \bar{v}_{kk'i}^2 \rangle = \frac{1}{N_k}\sum_{j\in\Omega_k} \bar{v}_{kk'ij}^2 \tag{12}$$

SEA parameter identification is performed by solving the SEA inverse problem relating $\langle \bar{\mathbf{V}}^2 \rangle$ to SEA loss matrix **L** through the normalized SEA power balanced equations.

$$\mathbf{I}\cdot Y/\omega_c = \mathbf{L}\cdot m\langle \bar{\mathbf{V}}^2 \rangle \Rightarrow \mathbf{L} = \left[m\langle \bar{\mathbf{V}}^2 \rangle\right]^{-1*}\mathbf{I}\cdot Y/\omega_c \tag{13}$$

m is the subsystem mass vector and * indicates the pseudo-inverse. In practice with

$$m = \mathcal{N}/4Y \tag{14}$$

previous equation is reshaped for direct solve of modal density vector $\mathcal{N}$. It leads to the local modal energy matrix power balanced equations given by:

$$\mathbf{I}/\omega = \mathbf{L}\mathcal{N}\boldsymbol{\varepsilon} = \mathcal{L}\boldsymbol{\varepsilon} \tag{15}$$

with elements of $\boldsymbol{\varepsilon}$ given by:

$$e_{kk'i} = \frac{1}{N_k}\sum_{j\in\Omega_k}\frac{\bar{v}_{kk'ij}^2}{4y_i y_j} \tag{16}$$

and

$$\mathcal{L} = \begin{bmatrix} \eta_1 \mathcal{N}_1 + \sum_{k'} \eta_{1k'} \mathcal{N}_{k'} & .. & -\eta_{N1}\mathcal{N}_N \\ .. & .. & .. \\ -\eta_{1N}\mathcal{N}_1 & .. & \eta_N \mathcal{N}_N + \sum_{k'} \eta_{k'N} \mathcal{N}_{k'} \end{bmatrix} \tag{17}$$

$\boldsymbol{\varepsilon}$ has dimension of modal energy and leads to accurate identification of $\mathcal{L}$ thanks to SEA+ algorithm that performs auto-partitioning into weakly coupled regions.

In practice, with lossless junctions, previous system is solved for identifying separately modal density and CLF. Model quality is assessed by comparing $\langle \tilde{V}^2 \rangle = 4Y^T \mathcal{L}^{-1*} Y / \omega$ with direct $\langle \bar{\mathbf{V}}^2 \rangle$ FRF input. Difference $\tilde{V}^2 - V^2$ gives the reconstruction error matrix plot as reconstruction performance index in SEA+ [15].

The MS-VSEA patch method is a variant formulation of the inverse SEA problem where the auto-partition is applied to pre-defined group of nodes instead of nodes. This method is thus providing a partition per frequency band corresponding to a specific grouping of patches into subsystems. Main advantage is to accurately reconstruct FE transfers over the whole frequency range. Figure 1 shows the flow chart of the VSEA process.

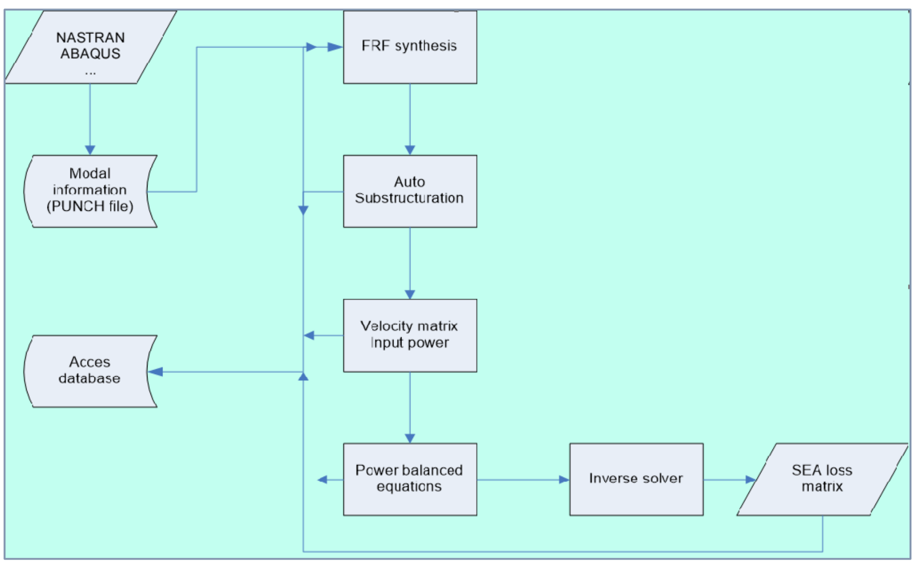

Figure 4. VSEA data flow

VSEA Modeling of a Car Component

A car cockpit is analyzed with VSEA and related subsystems are shown in Figure 5. The cockpit is made of various imbricated plastic parts embedded in a metallic support with various section properties. 500 nodes map the system. Starting from a FEM model of the component, around 2500 eigenvalues are extracted and modal amplitudes at all reference nodes are stored. SEA+ performs the FRF synthesis and numerical MS-VSEA model is generated from it.

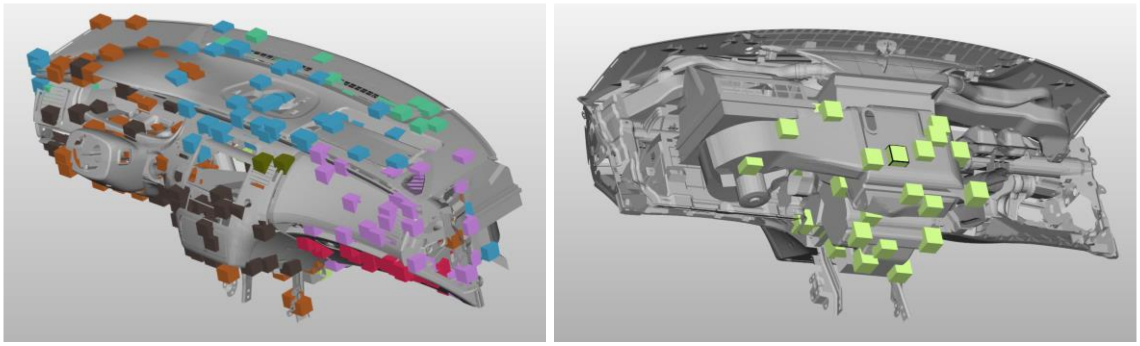

Figure 5. Left-Cockpit with identified nodal VSEA subsystems at 700 Hz and right-air-conditioning block identified as a subsystem

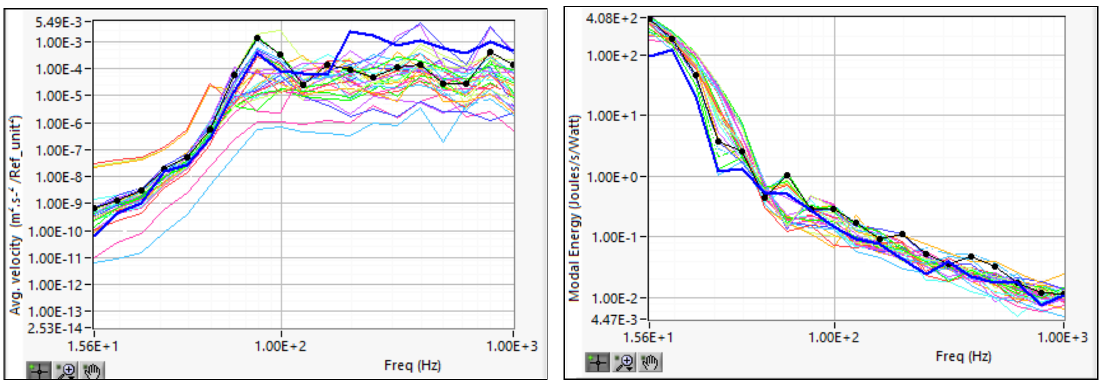

Figure 6. Left-V/F transfer velocity for a given excitation node in air-conditioning block subsystem and right-related local modal energy transfer as given by (16)

Effective subsystem domains are varying with frequency. Only 4 effective subsystems are found at 100 Hz, 7 at 250 Hz, 11 at 500 Hz and 13 at 1000 Hz. Detection of subsystems is performed on the matrix made of elements calculated from (16) that provides much less node-to-node scattering of nodal responses that the classical FRF (V/F) as shown in Figure 6.

In SEA+ GUI, only the finest partition is displayed for easy expansion in HF range as FEM modal extraction was limited to 1000 Hz due to FE size. The performance index of the mode guaranties the MS-VSEA band-averaged transfers to be within 2 dB from related direct FEM calculation. The frequency limit of MS-VSEA model is called the transition frequency, f_t.

Below f_t and unlike standard SEA model, the nodal information is preserved in VSEA model, making possible to predict response in VSEA nodes and not only as a mean over subsystem domain. Correlation with measured data is easier.

An SEA expander is then allocated to each VSEA subsystems. The expander is a “classical” SEA subsystem of which parameters are derived from analytical theory in order to take over the calculation of SEA CLF and modal densities above f_t. Patches may be conveniently chosen by the user for easier modeling of analytical SEA expanders by ascertaining a patch as a group of FE with same section property for example.

Above f_t both junction CLF and subsystem modal densities are analytically expanded to HF.

VSEA and Analytical Fluid/Structure Coupling

VSEA or MS-VSEA subsystems are coupled to analytical SEA cavities through a specific statistical radiation integral calculated by spatial windowing of an elementary infinite structural wave [14]. The related structural VSEA wavenumber is estimated from ratio of rotational over translational nodal principal-direction conductances.

For a subsystem of domain Ω_k,

$$\langle k \rangle = \sqrt{\frac{\langle Y_R \rangle_{\Omega_k}}{\langle Y_T \rangle_{\Omega_k}}} \tag{18}$$

Where $\langle Y_R \rangle$ and $\langle Y_T \rangle$ are respectively maximal local rotational and translational conductances averaged over Ω_k.

Adding Acoustic Trims to the VSEA Bare Model

All soft parts (internal trim panels, acoustic materials) are modeled as additional analytical subsystems or as attenuation spectra that filter the sound radiation. The latter are predicted by Transfer Matrix Method (TMM). Given an acoustic trim made of several layers (porous and/or elastic materials), related transmission (TL) and insertion (IL) losses are predicted by TMM under random incidence and infinite layer dimension. TL and IL are corrected for taking into account finite size of the trim. The base panels modeled as SEA subsystem are modified by trim presence (added mass and added damping).

Conclusions

SEA method provides a general theoretical framework for modeling both airborne and structure borne paths in complex industrial systems in the energy domain. We have brought to the fore some drawbacks of the method which have over years puzzled mechanical engineers in their understanding of SEA limitations. These limitations, mostly related to a priori sub-division of a system into regions satisfying SEA basic theoretical assumptions, have been overcome by introducing new concepts in the SEA modeling approach:

- Automated conversion of statistical dynamical information containing in a FEM model into SEA subsystems compatible with their analytical counterpart,
- Merging Energy Diffusion theory and Wave Transmission for improving coupling loss factor definition in region of strong coupling,
- Coupling Transfer Matrix Method which operates on 2D infinite layers with supporting SEA structures,
- Correcting calculation in infinite domain with the Spatial Windowing Technique.

Connecting all these features in a collaborative Graphical User Interface has helped in going further. Important topics, not covered in previous pages [16] are part of on-going research activities. They are progressively introduced in the SEA solver such as the non-resonant energy propagation in both structure and cavities which improves prediction of mass-controlled transfers of energy under acoustic and turbulent boundary layer loads.

Laminated shell damping and indirect mechanical coupling loss factors predictions are also an issue. Specific theories have been already stated and validation work is in progress.

References

[1] Lyon, R. H. and Maïdanik, G. (1962) Power Flow between Linearly Coupled Oscillators, *JASA* **34**, 623.

[2] Soize, C. (1993) A model and numerical method in the medium frequency range for vibroacoustic predictions using theory of structural fuzzy, *JASA* **94**(2), Pt 1, 849-866.

[3] Wilby, J. F. and Pope, L. D. (1979) Prediction of the acoustic environment in the Space Shuttle payload bay, *AIAA Conference*, Seattle, USA, REF AIAA 79-0643.

[4] Nefske, D.J. (1987) Power Flow finite element analysis of dynamic systems: basic theory and applications, *SAE*.

[5] Le Bot, A. (1994) *Equations énergétiques en mécanique vibratoire*, *Application au domaine des moyennes et hautes fréquences*, Thèse de l'école centrale de Lyon, France.

[6] Carcaterra, A. and Sestieri, A. (1995) Energy Density Equations and Power Flow in Structures, *JSV* 188(**2**), 269-282.

[7] Lalor, N. (1989) The Experimental Determination of Vibrational Energy Balance in Complex Structures, Paper 108429 Proc. *SIRA Conference on Stress & Vibration*, London, UK.

[8] Borello, G., Alliot and A. Kernilis A. (1991) Identification of SEA Coupling Loss Factors on a Liquid Rocket Engine, *Inter-Noise*, Sydney, Australia.

[9] Borello, G., Geoffroy, P. and Cuny, N. (1995) System Model of a High Speed Train Passenger Coach Using SEA and Prediction in Working Conditions, *Inter-Noise*, Newport Beach, CA, USA.

[10] Audoly, C. and G. Borello (2000) Feasibility of Experimental determination of SEA Damping Loss factors of submarine Structures, *Inter-Noise*, Nice, France.

[11] Gagliardini, L., L. Houillon,. Petrinelli, L. and Borello, G. (2003) Virtual SEA: mid-frequency structure-borne noise modeling based on Finite Element Analysis, *SAE-NVC* 2003-01-1555, Traverse City, MI, USA.

[12] Borello, G., Gagliardini, L., Houillon, L. and Petrinelli, L. (2005) Virtual SEA-FEA-Based Modeling of Structure-Borne Noise, *SVM*, January.

[13] Gagliardini, L., Thenail, D. and Borello, G. (2007) Virtual SEA for noise prediction and structure borne sound modelling, *Rieter Automotive Conference*, Pfaffikon, Switzerland.

[14] Villot, M., Guigou-Carter, C. and Gagliardini, L. (2001) Predicting the acoustical behaviour of finite size multi-layered structures by applying spatial windowing on infinite structures, *JSV* **245**, Number 3, 433-455(23).

[15] *SEA+ User-Guide* version from 2014 (InterAC SARL)

[16] G. Borello, G. (2016) Evolution des méthodes de calcul vibroacoustique des systèmes industriels en fonction de la fréquence, *Congrès Français d'acoustique*, Le Mans, France.

[17] Lyon, R. H. (1995) Statistical Energy Analysis and structural fuzzy, *JASA* **97**, Number 5 Pt1.