The traffic jerk for the full velocity different car-following model

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Abstract

Considering sudden change in vehicle's acceleration, an improved car-following model with a feedback control signal jerk was studied and presented in this paper. Stability analysis of the modified model was achieved according to the control theory method. Through theoretical analysis, the modified model may provide insights for developing management strategy to improve traffic jams.

Keyword: car-following models, feedback control, traffic jerk

1. Introduction

Traffic jams have been studied by many traffic simulation models namely, the car following models, the hydrodynamic models, the cellular automation models and the gas kinetic models [1-15]. In 1999, Konishi et al. [16] put forward a chaotic car-following model by setting the time delay feedback control, and researched single-lane traffic operation without reverse phenomenon under an open boundary condition. In 2006, Zhao et al. [17] put forward a control method for the suppression of the traffic jam. They gave a control signal which included the effect of velocity difference between the preceding and the considered vehicle. In 2007, Han and Ge [18] presented a coupled map car-following model for traffic flows with the consideration of the application of intelligent transportation systems. The control signal uniform to the velocity difference between the i-th vehicle in front and the (i+1)-th vehicle, and the developed model can improve the stability of traffic flow. Other research was connected with the control signal has been carried out recently [19, 20, 21, 22].

In 1961, Newell [2] put forward a car-following model with a differential equation and give graphic description of the optimal velocity (OV) function. In 1995, Bando et al. [3] proposed optimal velocity model (OVM) for car-following model. In the OVM, the acceleration of the n-th vehicle at time is identified by the difference between the actual velocity and an optimal velocity, which depends on the headway to the car in the front. In 2001 Jiang et al. [23] presented full velocity different model for car-following theory (FVDM) by considering both negative and positive velocity difference, which can give a better description of starting process than OVM. In 2012, Yu et al. [24] proposed a full velocity difference and acceleration model (FVDAM).The following cars in FVDAM react more quickly than those in FVDM and the stability of FVDAM is more stable than that of FVDM. However, no studies have ever tried to consider control method with jerk for the FVDM. Based on previous work, this paper investigates a new control scheme considering jerk. As we known, the vehicle's velocity changes are its acceleration, which means how quickly the vehicle increases and loses speed. Furthermore, abrupt change in vehicle's acceleration is called 'jerk', and it will affect the stability of traffic flow, so FVDM with the traffic jerk is studied in this paper.

In section 2, the FVDM is recovered and stability analysis is carried out. In section 3, the car-following model including a feedback control signal is put forward and the feedback control method is used to analyze the stability conditions. Conclusions are given in section 4.

2. Car-following model and its stability analysis

2.1. Full velocity different model

The dynamic equations of FVDM [23] are given by:

$$\begin{cases} \dfrac{d^2 x_n(t)}{dt^2} = a\left[V^{OP}(y_n(t)) - v_n(t)\right] + \lambda \Delta v_n(t), \\ \dfrac{dy_n(t)}{dt} = v_{n+1}(t) - v_n(t), \end{cases} \tag{1}$$

where $a = 1/\tau$ is the sensitivity of a driver, $y_n(t) = x_{n+1}(t) - x_n(t)$ and $\Delta v_n(t) = v_{n+1}(t) - v_n(t)$ are the headway and the velocity difference between the n-th considering vehicle and the preceding one, and $V^{OP}(y_n(t))$ is the optimal velocity function, which is written as follows:

$$V^{OP}(y_n(t)) = \frac{v_{\max}}{2}[\tanh(\Delta x_n(t) - h_c) + \tanh(h_c)], \tag{2}$$

where h_c is the safety headway distance.

2.2. Stability analysis

We assume that the leading vehicle runs constantly at speed v_0, so the steady state of the following vehicles are

$$(v, y) = (v^*, y^*). \tag{3}$$

Then, consider an error system around steady state (1), that is,

$$\begin{cases} \dfrac{dv_n^o(t)}{dt} = a\left[\Lambda y_n^o(t) - v_n^o(t)\right] + \lambda \Delta v_n^o(t), \\ \dfrac{\mathrm{d}y_\mathrm{n}^o(t)}{dt} = v_{n+1}^o(t) - v_n^o(t), \end{cases} \tag{4}$$

where $\Lambda = \dfrac{\partial \mathrm{V}^{\mathrm{OP}}(y_n^o(t))}{\partial y_n(t)}\Big|_{y_n(t)=y_n^o(t)}$, $\Delta v_\mathrm{n}^o(t) = v_{n+1}^o(t) - v_n^o(t)$, $v_n^o(t) = v_n(t) - v^*$, $\mathrm{y}_\mathrm{n}^o(t) = y_n(t) - y^*$.

The Laplace transformation for Eq. (4) leads to

$$\begin{cases} s\mathrm{V}_{\mathrm{n}}(s) - V_n(0) = a\left[\Lambda Y_n(s) - V_n(s)\right] + \lambda \Delta V_n(s), \\ sY_n(s) - Y_n(0) = V_{n+1}(s) - V_n(s), \end{cases} \tag{5}$$

where $\mathrm{V}_{\mathrm{n}}(s) = L(\delta v_n(t))$, $\mathrm{Y}_{\mathrm{n}}(s) = L(\delta y_n(t))$, $\mathrm{L}(\cdot)$ denotes the Laplace transformation, s is a complex variable. Form Eq. (5), we have

$$\mathrm{V}_{\mathrm{n}}(s) = \frac{a\Lambda + \lambda s}{s^2 + (a+\lambda)s + a\Lambda} V_{n+1}(s) + \frac{a\Lambda}{s^2 + (a+\lambda)s + a\Lambda} Y_n(0) + \frac{s}{s^2 + (a+\lambda)s + a\Lambda} V_n(0) \tag{6}$$

Let $p(s) = s^2 + (a+\lambda)s + a\Lambda$ and the transfer function can be obtained as

$$G(s) = \frac{a\Lambda + \lambda s}{s^2 + (a+\lambda)s + a\Lambda} \tag{7}$$

Based on stability theory, the traffic jam will never occur in the traffic flow system as long as the characteristic function $p(s) = s^2 + (a+\lambda)s + a\Lambda$ is stable and $\|G(s)\| \le 1$.

In order to make $p(s)$ stable, that $a > 0$ and $a\Lambda > 0$ should be confirm. According to the Hurwitz stability criterion, the OV function is monotonic increase (i.e. $\Lambda > 0$) and $a > 0$, we obtain that $p(s)$ is stable.

Then, we consider $\|G(s)\| \le 1$ which can be expressed as

$$|G(j\omega)|^2 = |G(j\omega)G(-j\omega)| = \frac{(a\Lambda)^2 + (\lambda\omega)^2}{(a\Lambda - \omega^2)^2 + (a+\lambda)^2\omega^2} \le 1. \tag{8}$$

The sufficient condition can be obtained as

$$\omega^4 + a^2\omega^2 - 2a\Lambda\omega^2 + 2a\lambda\omega^2 \ge 0, \omega \in [0, +\infty), \tag{9}$$

which can be rewritten as

$$\lambda \ge \Lambda - \frac{a}{2}. \tag{10}$$

If the condition $\lambda \ge \Lambda - \frac{a}{2}$ is satisfied, the traffic system will be stable.

3. Control scheme

The aim of this paper is to purpose a control scheme for suppression of congested traffic in the car-following model. A feedback control signal $u_n(t)$ is designated as follows:

$$u_n(t) = k\left(\frac{dv_n(t)}{dt} - \frac{dv_n(t-1)}{dt}\right) \tag{11}$$

where k is the feedback gain, which can be adjusted. The control signal term is added to Eq. (1) as

$$\begin{cases} \dfrac{d^2 x_n(t)}{dt^2} = a\left[V^{OP}(y_n(t)) - v_n(t)\right] + \lambda \Delta v_n(t) + u_n(t), \\ \dfrac{dy_n(t)}{dt} = v_{n+1}(t) - v_n(t), \end{cases} \tag{12}$$

The control signal $u_n(t)$ is traffic jerk.

Similarly, we assumed that the leading vehicle runs with constant speed v_0, the steady state of the following vehicles are the same of Eq.(3). Then, consider an error system around steady state (12), that is

$$\begin{cases} \dfrac{dv_n^0(t)}{dt} = a\left[\Lambda y_n^0(t) - v_n^0(t)\right] + \lambda \Delta v_n^0 + u_n^0(t), \\ \dfrac{dy_n^0(t)}{dt} = v_{n+1}^0(t) - v_n^0(t), \end{cases} \tag{13}$$

where $\Lambda = \dfrac{\partial V^{OP}(y_n^0(t))}{\partial y_n(t)}\Big|_{y_n(t)=y_n^0(t)}$, $v_n^0(t) = v_n(t) - v^*$, $y_n^0(t) = y_n(t) - y^*$,

$u_n^0(t) = k\left[\dfrac{dv_n^0(t)}{dt} - \dfrac{dv_n^0(t-1)}{dt}\right]$.

The Laplace transformation for Eq. (13) leads to

$$\begin{cases} sV_n(s) - V_n(0) = a\left[\Lambda Y_n(s) - V_n(s)\right] + \lambda \Delta V_n(s) + U_n(s) \\ sY_n(s) - Y_n(0) = V_{n+1}(s) - V_n(s) \end{cases} \tag{14}$$

where $U_n(s) = k\left[(sV_n(s) - V_n(0)) - e^{-s}(sV_n(s) - V_n(0))\right]$, $V_n(s) = L(\delta v_n(t))$,

$Y_n(s) = L(\delta y_n(t))$, $L(.)$ denotes the Laplace transformation, s is a complex variable. Form Eq. (14), we have

$$\begin{aligned} V_n(s) &= \frac{a\Lambda + \lambda s}{a\Lambda + (a+\lambda)s + s^2 - k(1-e^{-s})s^2} V_{n+1}(s) + \frac{a\Lambda}{a\Lambda + (a+\lambda)s + s^2 - k(1-e^{-s})s^2} Y_n(0) \\ &= \frac{\left[1 - k(1-e^{-s})\right]s}{a\Lambda + (a+\lambda)s + s^2 - k(1-e^{-s})s^2} V_n(0) \end{aligned} \tag{15}$$

Let $1 - e^{-s} = s$, substituting it into Eq. (15), which leads to

$$V_n(s) = \frac{a\Lambda + \lambda s}{a\Lambda + (a+\lambda)s + s^2 - ks^3} V_{n+1}(s) + \frac{a\Lambda}{a\Lambda + (a+\lambda)s + s^2 - ks^3} Y_n(0) + \frac{(1-ks)s}{a\Lambda + (a+\lambda)s + s^2 - ks^3} V_n(0) \tag{16}$$

Let $p^*(s) = a\Lambda + (a+\lambda)s + s^2 - ks^3$ and the transfer function can be obtained as

$$G^*(s) = \frac{a\Lambda + \lambda s}{a\Lambda + (a+\lambda)s + s^2 - ks^3} \tag{17}$$

Thus, traffic jams will never occur in the traffic flow system if $p(s)$ is stable and $\left\| G^*(S) \right\|_\infty \le 1$. Similarly to the second part of the analysis, the sufficient condition is given as

$$\left| G^*(j\omega) \right|^2 = \left| G^*(j\omega) G^*(-j\omega) \right| = \frac{(a\Lambda)^2 + (\lambda\omega)^2}{(a\Lambda - \omega^2)^2 + \left[(a+\lambda) + k\omega^2 \right]^2 \omega^2} \le 1 \qquad (18)$$

Then, we can obtain the sufficient condition through the above analysis, that is

$$k^2\omega^4 + \left[2(a+\lambda)k + 1 \right]\omega^2 + a(a + 2\lambda - 2\Lambda) \ge 0 \, , \omega \in [0, +\infty) \qquad (19)$$

The sufficient condition for Eq. (19) is

$$\begin{cases} (2\lambda k + 1)^2 + 4ak(1 + 2k\Lambda) \le 0, if \ \dfrac{2(a+\lambda)k + 1}{2k^2} < 0 \\ a(a + 2\lambda - 2k) \ge 0, if \ \dfrac{2(a+\lambda)k + 1}{2k^2} > 0 \end{cases} \qquad (20)$$

4. Summary

In this paper, a new feedback control signal ‘jerk’ is added to FVDM. The stability condition of developed model is analyzed by using feedback control theory. Through theoretical analysis, the range of reaction parameter λ for the model with and without feedback control signal obtained.

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