

Hull form optimization based on a NM+CFD integrated method for KCS

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Abstract

It is a definite trend and hot topics of hull form optimal design based on computational fluid dynamics(CFD). Hull form optimization is carried out in this paper which combines the Neumann-Michell (NM) theory with CFD technology (NM+CFD integrated method) to OPTShip-SJTU, an optimization tool. The Free Form Deformation (FFD) method adopted for automatically modifying the hull form are illustrated. In order to reduce the overall highly computational effort, not only the surrogate model is established based on the samples produced by OLHS method and is used to directly predict the total resistance in optimization process, but also a NM+CFD integrated method, the NM theory for evaluating wave resistance and CFD technology based on RANS for evaluating viscous resistance of double body, are discussed to evaluate the total resistance of ships. In addition, NSGA-II, a multi-objective genetic algorithm, is implemented to produce pareto-optimal front. In the present paper the KRISO 3600TEU container ship model (KCS) is chosen as initial ship and optimal solutions with obvious total resistance coefficient reductions at specific speeds(at $Fr=0.2, 0.26$) are obtained. Eventually, one typical optimal hull is analyzed by a RANS-based CFD solver naoe-FOAM-SJTU. Numerical results confirm the availability and reliability of this multi-objective optimization tool.

Keywords: Hull form optimization, total resistance coefficient, FFD, OPTShip-SJTU, naoe-FOAM-SJTU solver.

Introduction

Ship designers often design a new ship mostly by their own experience in accordance with the requirements proposed by shipping companies[1]. Generally, designers can attempt to transform several initial ships with similar usage, similar shapes and as well as with satisfaction of ship owners during the operation and then predict and check performances of the new ship over and over. The above design process is a single-threaded circle, which mainly depends on designers' experience and intuition.

With the development of computer technologies and computational fluid dynamics(CFD), ship optimization design has raised the interest of researchers and designers, which is a converse process absolutely different from the traditional ship design process mentioned above. It is a process where to achieve the best performances of a new ship directly drives ship design. During the last several decades, a rapidly increasing number of papers devoted to ship optimization design based on hydrodynamic performance have been yielded with the advantage of optimization techniques and high-performance computer(HPC), resulting in the huge development of ship design[1]-[8]. Among these papers, the initial ships often adapted are Wigley[9][10], an internationally common and mathematical ship type, and S60, both with simple hull forms and a great quantity of experimental data. However, the increasing complexity of a real-life optimization problem in ship industry has raised the challenges for designers[], because hull form optimization of a complex geometry typically involve a large number of variables, different disciplines and conflicting objectives, requiring hundreds or thousands of function evaluations to converge to an optimal design .

Thanks to the development of some excellent modern optimization algorithms[11]-[14] such as the Non-dominated Sorting Genetic Algorithm-II (NSGA-II) and particle swarm optimization (PSO), multi-objective optimization of ship hulls makes a significant breakthrough[17], and ongoing research is still much concerned about this topic[18][30].

Actually, how to quickly and accurately evaluate objective functions or the hydrodynamic performance during the optimization process is an important segment. Both of the potential-flow theory and the advanced RANS-based CFD method had been employed to predict the hydrodynamic performance during the hull optimization. If high-fidelity solvers based on CFD are used as analysis tools (e.g., RANS solvers), many conditional optimization methods become more and more expensive. However, the potential-flow theory can be used in evaluating the wave-making resistance in calm water because of the efficiency[31][32], and a RANS-based CFD method can be just used as predicting the viscous resistance with a double-model. Furthermore, the total resistance can be expressed as the sum of the wave-making resistance and viscous resistance[19].

In the present paper, KCS is chosen as the initial hull form to locally optimize its bow and its stern, respectively, based on the minimum total resistance coefficients at two specific speeds. First, the Design of Experiment is used to select a reasonable optimal design space. Specifically, optimized Latin hypercube sampling (OLHS) method is applied here which satisfies the requirements of orthogonality and uniformity to obtain different design variables, that is to say, different ship samples. Then, these ship samples are deformed by free form deformation(FFD). Next step is to evaluate their total resistances at two specific speeds, so called objective functions, where wave-making resistances are evaluated by NM theory and viscous resistances are evaluated by CFD-based naoe-FOAM-SJTU solver. So far, the surrogate model [23] is adopted to describe the complex relationship between the design variables and multi-objective functions, which largely decreases the optimization difficulty and computational cost. Last but not least, a vital multi-objective optimization process is completed by NSGA-II, a series of optimal ship hull obtained. The whole optimization frame can be seen as follows (Fig. 1) .

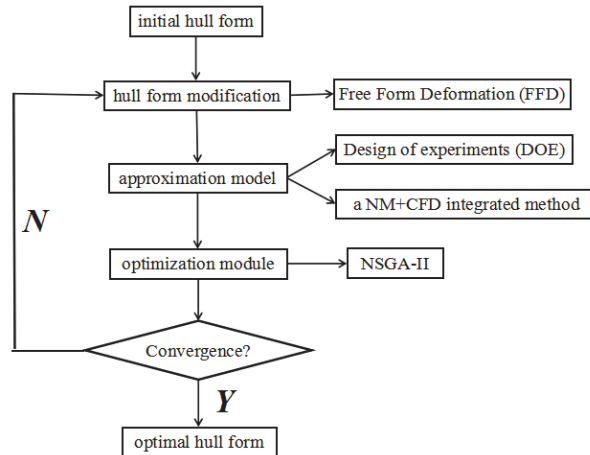


Figure 1. The flow chart of the iterative optimization process

Hull form deformation

An effective and rational method for hull form deformation is indispensable and crucial in the optimization process of ship design. One Hull form should be quickly and reasonably transformed to another new one. And there should be as less as possible deformation parameters involved in the optimization design, otherwise it will increase the complexity of the problem and lead to vast computational cost in multiples. Here FFD method is chosen to modify hull form locally, based on the idea of enclosing the ship within a cube, and transforming the hull form within the cube as the cube is deformed. FFD method was first

described by Thomas W. Sederberg and Scott R. Parry in 1986[21][22], and was based on an earlier technique by Alan Barr[20]. It is widely used in the optimization of ships, because less design variables related are involved in the optimization, surfaces are flexibly transformed, it is easily realized by making a program and so on. It is strongly dominant among many deformation methods that the main dimension of the initial ship can be limited and any new shape obtained by FFD method can be more reasonable and practical. In this paper, FFD method is applied to locally modify the bulb bow and the stern of KCS, respectively.

Total resistance evaluation

The total resistance of ships can be solved according to two methods of division. One is according to the assumption of Froude, through his experiments, Froude realized that the ship resistance problem had to be broken into two different parts: residuary resistance (mainly wave making resistance) only related to Froude number (Fr) and frictional resistance only related to Reynolds number (Re). However, the influence of the two parts is ignored by using this method. Actually, especially for the fat full ship type, ΔC_f will be negative.

So in 1950s, Hughes proposed another method—three dimensional conversion, which was recommended as the standard conversion at ITTC in 1978. Through this conversion, total resistance (R_t) is broken into two new parts: wave-making resistance (R_w) related to Froude number and the viscous resistance (R_v) (the sum of the viscous pressure resistance and friction resistance) related to Reynolds number.

$$R_t = R_w + R_v \quad (1)$$

In this paper, the above standard conversion is chosen to predict the total resistance, wave-making resistance calculated by NM theory and viscous resistance calculated by simulating the flow field around the double ship model based on RANS, which is abbreviated as the NM+CFD integrated evaluation. Nobless et al. [23] present an efficient potential theory, Neumann-Michell (NM) theory, which provides more accurate prediction of wave-making resistance and wave profiles than the Hogner slender-ship approximation, with no appreciable increase in computational cost (seconds on a PC) for the classical Wigley parabolic hull. Besides, there are lots of research about comparison of experimental measurements of wave-making resistance with numerical predictions obtained using a preliminary version of the NM theory for the Wigley hull, the Series 60 and DTMB 5415 model[24]-[26]. A RANS-based CFD solver naoe-FOAM-SJTU, which is developed under the framework of the open source code, OpenFOAM, and has been validated in computation of a ship with heave and pitch motion in head waves[18].

The validation study for the NM+CFD integrated method is carried out before the optimization. For KCS, the results calculated by naoe-FOAM-SJTU and the NM+CFD integrated method and experimental data are respectively shown in Table 1.

Table 1. Total resistance coefficients predicted by the NM+CFD integrated method, CFD and experimental data.

Comparison \ Speed		Fr=0.2	Fr=0.26
Ct	NM+CFD	3.72E-03	3.82E-03
	CFD	3.58E-03	3.84E-03
	EXP	3.46E-03	3.75E-03
Deviation	NM+CFD-EXP	-7.09%	-1.74%
	CFD-EXP	3.47%	2.40%

As shown in Tab. 1, the results based on the NM+CFD integrated method are within the error allowed, (-7.09% at $Fr=0.2$ and -1.74% at $Fr=0.26$), which is a little bit worse than the results totally based on CFD. Even so, this integrated method is still worth to be adopted because of its lower computational time cost. It's a huge advantage for hull form optimization design.

The definition of multi-objective optimization

Multi-objective optimization problem is a problem of multiple criteria decision making, that is concerned with mathematical optimization problems involving more than one objective function to be optimized simultaneously. Multi-objective optimization problem has been applied in many fields of science, including engineering, economics and logistics where optimal decisions need to be taken in the presence of trade-offs between two or more conflicting objectives.

In mathematical terms, a multi-objective optimization problem can be formulated as

$$\begin{aligned} \min & (f_1(x), f_2(x), \dots, f_k(x)) \\ \text{s.t.} & x \in X \end{aligned}$$

Where the integer $k \geq 2$ is the number of objectives and the set X is the feasible set of decision vectors.

In the ship industry, there is still a problem about the trade-offs between each performance of a new ship during the ship design process. The following content will clearly describe a complete multi-objective optimization of ship design.

The establishment of the optimization problem

For an entire optimization problem to be solved, the following basic items must be specified in detail: (1)an initial hull form to be optimized and the region(s) to be modified;(2)the objective function to be minimized and the design variables to be used;(3)the constraints to be defined. All of these items will be described in terms of the ship optimization presented by this paper.

Initial hull form

The initial hull form is the KRISO 3600TEU container ship model (KCS), which was conceived to provide data for both explication of flow physics and CFD validation for a modern container ship with bulb bow and stern. There is a large experimental database for KCS due to an international collaborative study on experimental/numerical uncertainty assessment between NMRI, MOERI and SVA[29].

The geometry of the initial model is presented in Fig.8 and the principal dimensions of KCS in table 2.



Figure 2. The geometry of KCS

Table 2. The principal dimensions of KCS

Principal Dimensions	full-scale ship	ship model
Length between perpendiculars L_{pp}/m	230	7.28

Length of waterlines L_{wl}/m	232.5	7.36
Breadth moulded B/m	32.2	1.019
Depth moulded D/m	19	0.6013
Draught T/m	10.8	0.3418
Block coefficient C_b	0.651	0.651

Multi-objective function and design variables

The multi-objective functions to be minimized is the total resistance coefficient of KCS sailing in calm water at two speeds of $Fr=0.2$, $Fr=0.26$. This condition corresponds to using a reference length of 7.36m, that is the length of the ship's model used in the experimental validation.

$$C_t = C_w + C_v \quad (2)$$

$$C_w = \frac{R_w}{0.5\rho U^2 S} \quad (3)$$

$$C_v = \frac{R_v}{0.5\rho U^2 S} \quad (4)$$

The deformation region is only the foremost part of the ship ($x=3.45\sim 3.99m$) and the stern of the ship ($x=-3.44\sim -0.44$), with the origin of coordinates at the midship in Fig. 4. As explained in the introduction, this is the typical redesign problem of some part of an existing complex system, a necessity which often arises in real industrial applications. At the stern of the ship, two control boxes are used in order to modify the origin shape of the stern to any practical new one. In Fig. 5, some certain movable control points and the other fixed control points are clearly grouped into two kinds of colors, red and green.

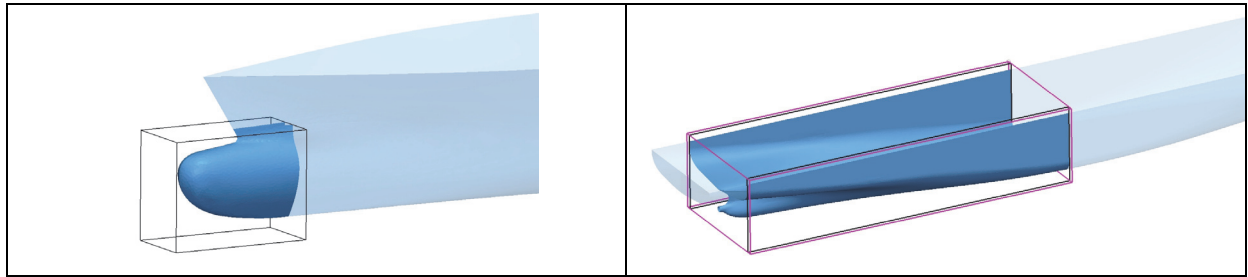
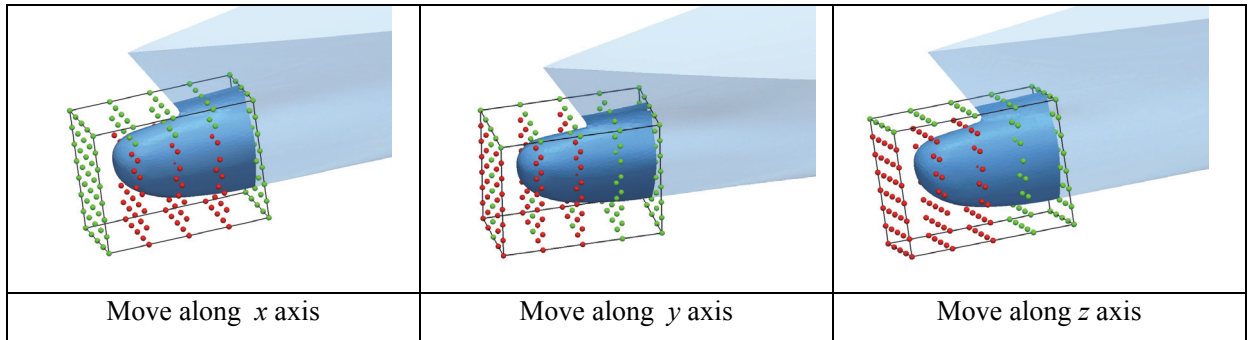


Figure 3. The modification regions by FFD method



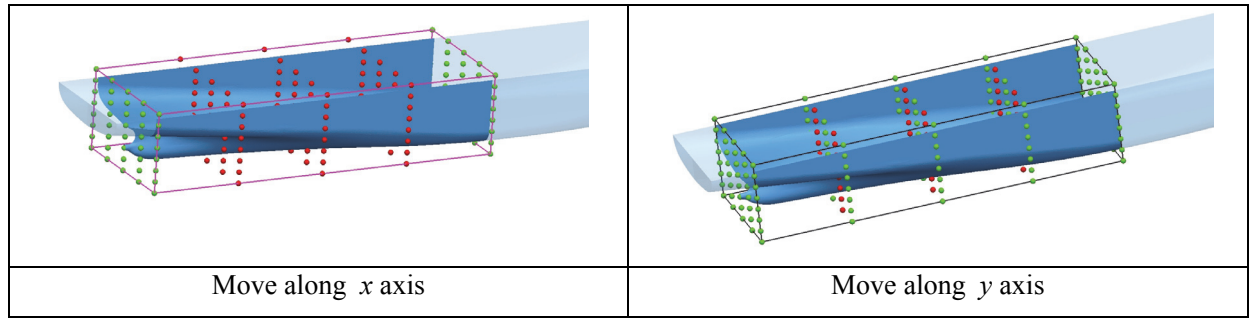


Figure 4. The modification regions by FFD method

Additionally, some geometric constraints are imposed on the design variables, the displacement (∇), the wetted surface area (S_{wet}) and the principal dimensions of the ship. Detail information regarding these constraints is reported in Tab. 4.

Table 3. Definition of the optimization problem

Type	Definition	Note
Initial hull	the KRISO 3600TEU container ship model (KCS)	
Objective functions	$f_{obj}^1 = C_t = C_w + C_v, \quad \text{at } Fr = 0.2$ $f_{obj}^2 = C_t = C_w + C_v, \quad \text{at } Fr = 0.26$	Bare hull Aim is to search for hull forms with potential drag reduction at given speeds
Design variables		
Δx_1 (Variable1)	[-0.0736, 0.0736]	Displacement in x direction in the fore-part region
Δy_1 (Variable2)	[-0.0368, 0.0368]	Displacement in y direction in the fore-part region
Δz_1 (Variable3)	[-0.04784, 0.04784]	Displacement in z direction in the fore-part region
Δx_2 (Variable4)	[-0.05152, 0.05152]	Displacement in x direction in the aft-part region
Δy_2 (Variable5)	[-0.0736, 0.08832]	Displacement in y direction in the aft-part region
Geometric constraints		
Main dimensions	L_{pp} , D , B are fixed	
Displacement (∇)	Maximum variation $\pm 1\%$	
Wetted surface area (S_{wet})	Maximum variation $\pm 1\%$	
Experimental design	OLHS method	Generate 100 sample points
Approximation model	Kriging model	
Optimizer	NSGA-II	
Size of population	200	
Number of generations	300	

Numerical results: the optimal design

Based on the optimal Latin hypercube design method (OLHS), 100 sample points about five design variables Δx_1 (displacement of control points in x direction in the fore part), Δy_1 (displacement of control points in y direction in the fore part), Δz_1 (displacement of control points in z direction in the fore part), Δx_2 (displacement of control points in x direction in the aft part), Δy_2 (displacement of control points in y direction in the aft part) are generated, then the corresponding values of multi-objective function, total resistance coefficients, are obtained using the NM+CFD integrated method.

Additionally, the ANOVA test is used to reflect the effects of each design variable on the objective functions. Denote by V1~V5 the five design variables (Δx_1 , Δy_1 , Δz_1 , Δx_2 , Δy_2) (see Fig. 6). The effects of design variables on different objective functions vary widely. V2(Δy_1) and V1(Δx_1) have main effects on f_{obj}^1 , while V2(Δy_1), V5(Δy_2) and V1(Δx_1) on f_{obj}^2 . But the total effect of the others is not neglected, and the computational cost considering all of five design variables is adequately affordable, thus, all of which are adopted in optimization.

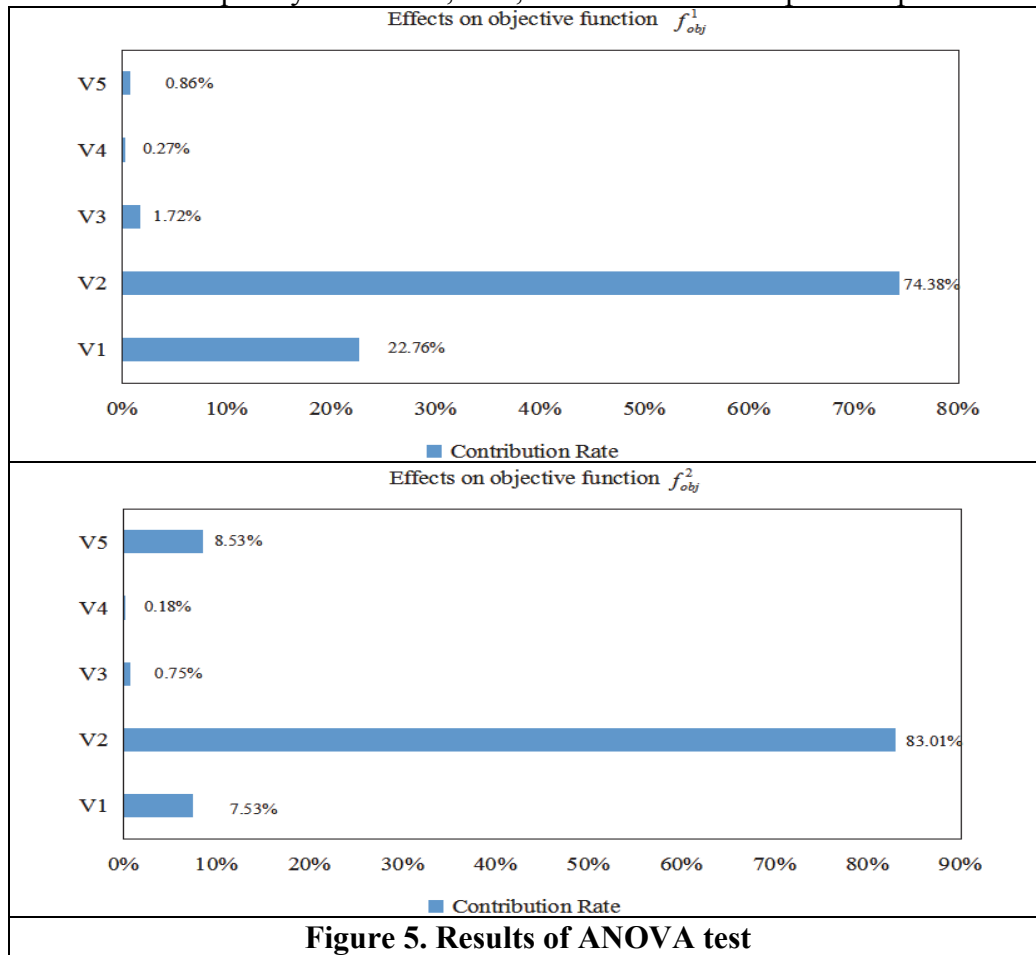


Figure 5. Results of ANOVA test

The Pareto front is reported in the function space in Fig. 7, where each red point represents an optimal solution while one typical example is marked in red to be analyzed further. Fig. 6 shows the Pareto optimal set from multi-objective optimization with NSGA-II algorithm. A reduction in resistance coefficients can be seen from Fig. 7. As an example of the Pareto optimal ships, one optimal configuration detected by the procedure is reported as the mark of the green point in Fig. 7.

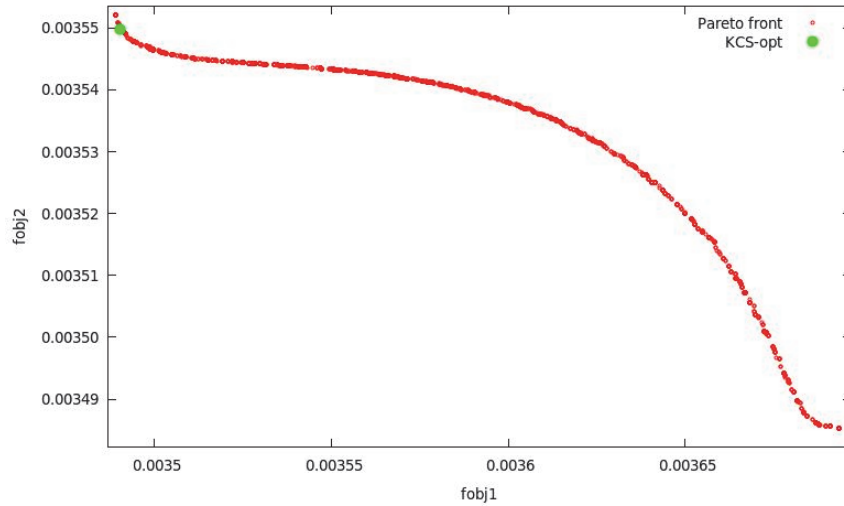


Figure 6. Pareto optimal points and optimal cases in objective functions space

Although the control modification regions are small, quite different configurations are readily yielded, all the Pareto optimal solutions, and different alternatives may be considered at this stage. KCS-opt represents the optimal hull form selected in this paper. As shown in Fig. 8 and Fig. 9, the bulb bow of the optimal hull form is evidently upturned than the initial one, and the stern lines are slightly changed, which corresponds directly with the ANOVA results presented before.

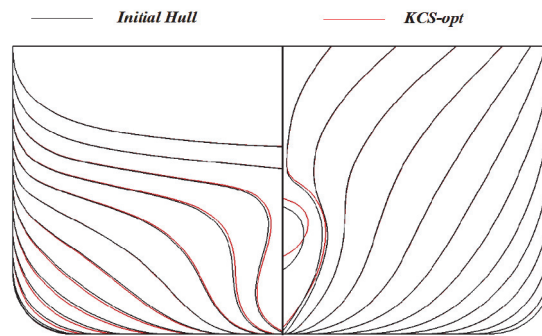


Figure 7. Body plans between the initial hull form and the optimal hull form

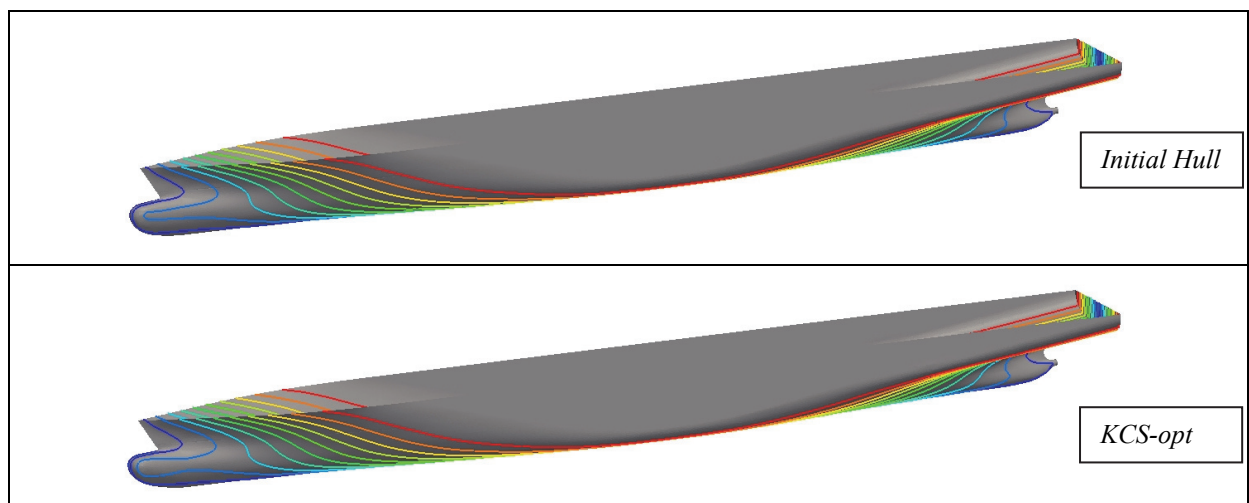


Figure 8. Buttock lines and 3D models between the initial hull form and the optimal hull form

The following table 4 shows the comparison of the results between the optimal hull form and the initial one. The respective reduction of the two total resistances is clearly seen, 4.73% reducing at $Fr=0.2$ and 8.32% reducing at $Fr=0.26$. However, there is a strange phenomenon that the total resistance of the KCS-opt at $Fr=0.26$ is even lower than at $Fr=0.2$. Further to understand, bulbous bow is first designed only to produce the positive effects on the resistance performance at the design speed. If so, it appears that the resistance is higher at other speeds.

Table 4. The prediction results for the initial and optimal hull forms based on NM+CFD integrated method

Comparison \ Speed		$Fr=0.2$	$Fr=0.26$
C_t	Initial Hull	3.72E-03	3.82E-03
	KCS-opt	3.55E-03	3.50E-03
Reduction		4.73%	8.32%

Validation with naoe-FOAM-SJTU solver

A high-fidelity numerical computation tool, naoe-FOAM-SJTU solver, is used to provide more accurate validation of the optimal hull form considering viscous effect, based on RANS method. Here, the total resistances between the initial hull form and KCS-opt mentioned above are only predicted at the design speed $Fr=0.26$. And the numerical results are presented in Tab. 5. KCS-opt displays a decrease of the total resistance coefficients of 3.39% at $Fr=0.26$.

Table 5. Numerical results for the initial and optimal hull forms by naoe-FOAM-SJTU solver

Design Speed		$Fr=0.26$
C_t	Initial Hull	3.84E-03
	KCS-opt	3.71E-03
Reduction		3.39%

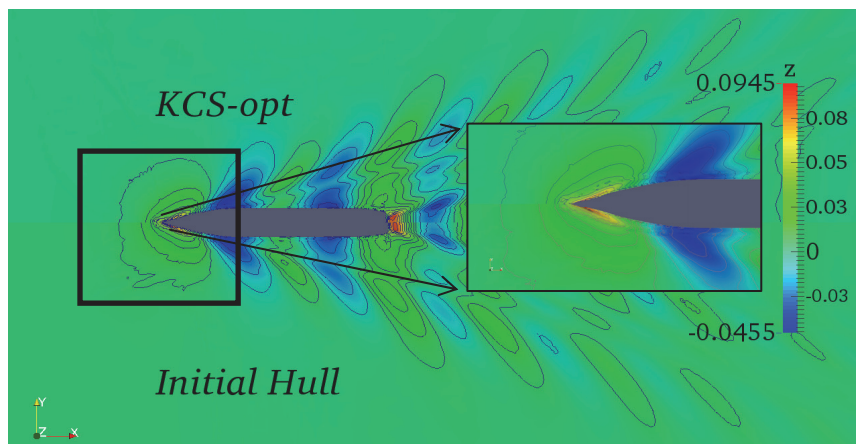


Figure 9. Wave patterns of free surfaces between the initial hull form and the optimal hull form

The computed wave patterns are reported in Fig. 8 The wave field caused by KCS-opt is with a smaller bow wave, a clear sign that the wave component of the ship's resistance has been reduced. A typical changes in the foremost wave pattern is enlarged, KCS-opt slightly reduces the amplitude of the bow wave. However, the wave pattern along the aft part of KCS-opt is a little bit changed, which is corresponding to the ANOVA test. It is partly illustrated that the multi-objective optimization, using OPTShip-SJTU solver, is reliable.

Conclusions

1. A numerical multi-objective optimization tool, OPTShip-SJTU, has been developed and tested in present work. the KRISO 3600TEU container ship model (KCS) is adopted as initial hull form, and the aim is to search for optimal hull forms with improved resistance performances at two given speeds ($Fr = 0.20, 0.26$).
2. During the procedure of optimization, the regions of bulb bow and stern are deformed with free-form deformation (FFD) method. FFD method is sufficiently flexible to generate a series of realistic alternative hull forms with a few number of design variables involved.
3. OPTShip-SJTU solver based on the integrated method of Neumann-Michell (NM) theory and Reynolds Average Navier Stokes (RANS) as the hydrodynamic performance evaluation module to predict the total resistance, turns out to be applicable for a real optimization problem.
4. The optimizer based on a multi-objective genetic algorithm, NSGA-II, and pareto-optimal front is obtained eventually.
5. The validation of the optimization problem is also carried out by naoe-FOAM-SJTU, a solver based on OpenFOAM source code. It shows the multi-objective optimization is acceptable and useful. and the results of OPTShip-SJTU solver should be further validated and verified by experimental data.

Acknowledgements

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