Computation and use of the Laurent series of the inverse in RBF-FD problems

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Abstract

In this presentation we describe a new approach to study Radial Basis Function (RBF) interpolation in the limit of increasingly flat basis functions. This new approach computes the Laurent series of the inverse of the RBF interpolation matrix using a semi-analytical algorithm based on the resolvent formalism used in complex analysis. The inputs to the algorithm are the matrix coefficients of the series expansion of the interpolation matrix in powers of the shape parameter. The outputs of the algorithm are the matrix coefficients of the Laurent expansion of the inverse, which are computed using recursive analytical formulae. Once the Laurent series is obtained, it can be successfully used to analyze several relevant issues related to RBF methods in the limit of increasingly flat basis functions:

1. Derivation of the limiting interpolation polynomials.
   These polynomials have been derived in some specific cases using symbolic language, but using the Laurent series we not only obtain the leading order polynomial but a series of polynomials in powers of the shape parameter. We derive these polynomials for any arbitrary node layout.
2. Computation of the optimal value of the shape parameter in RBF interpolation. We propose a new method that makes use of the first two terms of the series of polynomials to obtain the value of the shape parameter that minimizes the interpolation error.
3. Derivation of RBF finite difference (RBF-FD) formulas. We use the series of polynomials in powers of δ to obtain the weights of RBF-FD formulas. In this way, we obtain formulas for each weight as a series in powers of δ. We also use these weights to derive exact formulas for the local truncation error.

Keywords: Radial Basis Functions, Interpolation, Optimal shape parameter.