Reliability Analysis of Slope Stability using Monte Carlo Simulation and Comparison with Deterministic Analysis

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Abstract

Traditional slope stability analysis is limited to the use of single valued parameters to analyze a slope's characteristics. Consequently, traditional analysis methods yield single valued estimates for factor of safety of a slope's stability. However, the inherent variability of the soil characteristics which affect slope stability indicates that the stability of a slope is a probabilistic rather than a deterministic situation. In other words, the stability of a slope is a random process which is dependent on the relative distribution of controlling soil parameters. For a natural slope, the stability deciding parameters vary considerably throughout the extent of slope. In this paper, the variability of soil properties and their effect on stability of a natural slope has been studied incorporating the probabilistic analysis using Monte Carlo simulation and deterministic analysis using Geo-Studio and PLAXIS. The factors of safety have been determined using the two approaches and effect of dynamic loading input on slope stability has been studied.

Keywords: Slope stability, deterministic approach, probabilistic analysis Monte Carlo method.

Introduction

Slope instability is responsible for damage to public and private property every year. Slope failures can be manifested as landslides or by other slowly occurring processes such as soil seriously damaged or destroyed. Slope instability is a complex phenomenon that can occur at many scales and for many reasons. Slope stability analyses and stabilization require an understanding and evaluation of the processes that govern the behavior of slopes.

Real life failures in naturally deposited mixed soils are not necessarily circular, but prior to computers, it was far easier to analyze such a simplified geometry. Nevertheless, failures in 'pure' clay can be quite close to circular. Such slips often occur after a period of heavy rain, when the pore water pressure at the slip surface increases, reducing the effective normal stress and thus diminishing the restraining friction along the slip line. This is combined with increased soil weight due to the added groundwater. A 'shrinkage' crack (formed during prior dry weather) at the top of the slip may also fill with rain water, pushing the slip forward. At the other extreme, slab-shaped slips on hill sides can remove a layer of soil from the top of the underlying bedrock. Again, this is usually initiated by heavy rain, sometimes combined with increased loading from new buildings or removal of support at the toe (resulting from road widening or other construction work). Stability can thus be significantly improved by installing drainage paths to reduce the destabilizing forces. A weakness along the slip circle may remain at the reoccurrence of the next monsoon. If the forces available to resist movement are greater than the forces driving the movement, the slope is considered stable. Factor of safety is calculated by dividing the forces resisting movement by the forces driving movement. In earthquake-prone areas, the analysis is typically run for static conditions and pseudo-static conditions, where seismic forces from an earthquake are assumed to add static loads to the analysis.

The slope stability analyses are performed to assess the safe and economic design of human-made or natural slopes (e.g. embankments, road cuts, open-pit mining, excavations, landfills etc.) and the equilibrium conditions [1]-[3]. The term slope stability may be defined as the resistance of inclined surface to failure by sliding or collapsing. The main objectives of slope stability analysis are finding endangered areas, investigation of potential failure mechanisms, determination of the slope sensitivity to different triggering mechanisms, designing of optimal slopes with regard to safety, reliability and economics, designing possible remedial measures, e.g. barriers and stabilization. Successful design of the slope requires information about site characteristics, e.g. properties of soil/rock mass, slope geometry, alteration of materials by faulting, joint or discontinuity systems, movements and tension in joints, earthquake activity, etc. Choice of correct analysis technique depends on both site conditions and the potential mode of failure, with consideration being given to the varying strengths, weaknesses and limitations inherent in each methodology. The hypothesis of this research is that analysis of slope stability can be more methodological using the information about probability distribution of the slope's characteristics to determine the slope stability from the output of the analysis. Knowledge of the probability distribution of the output allows the engineer to assess the probability of slope failure. Therefore, an allowable risk criterion can be used to establish a consistent target for the design process [4].

Scope and Objectives

Stability of slopes, natural or man-made, is particularly important for any hill road. Disturbance to slope can occur due to erosion by rainfall and run-off and consequent slides. During monsoons the hill roads experience slips, erosions and major and minor landslides at many places. Check for the stability of the slopes is very necessary in order to ensure the stability of the slope as it would affect the life of people directly as landslide causing life loss and indirectly as the hindrance to flow of the traffic. Since the profile is along the National Highway 21, so its failure can cause the closing of the highway and it has been observed many times that it has closed previously. Rainy season causes the maximum disturbance in its stability. Hence, slope stability is vital for prevention of landslides/slips [5]. If the cut slopes are not properly designed, it will fail and would causes huge loss to mankind in a direct or indirect way. Taking into consideration above factors and importance of the stability, essential remedial measures are required and should be properly designed. Moreover consideration of various uncertainties involved in the properties of the soil which ultimately determine the stability of slope should be taken into account. For that purpose, statistical analysis or reliability analysis of slope becomes necessary and should be performed for a particular slope to check the reliability index of that particular slope. Reliability analysis of slope stability has attracted considerable research attention in the past few decades [6]-[10]. Reliability of slope stability is frequently measured by ''reliability index,'' and slope failure probability, P_f, which is defined as the probability that the minimum factor of safety (FS) is less than unity (i.e., $P_f = P\ (FS < 1)$). Various solution methods have been proposed to estimate P_f and Reliability Index. Among the most widely used methods are the first order second moment (FOSM) method, first order reliability method (FORM, also referred to as the Hasofer-Lind method) [11] and direct Monte Carlo simulation [12]. The objective of this research is to develop a probabilistic model for slope analysis by (a) understanding the concept of reliability analysis and its application in slope stability analysis, (b) performing the reliability analysis of slope stability using Monte Carlo simulation (using

RiskAMP [13]; and Geo5 [14], (c) performing the slope stability analysis with the help of PLAXIS [15] and (d) comparing the results obtained from different methods.

Methodology

The methodology include the preparation of the contour map of the slope to determine the geometry and assessing the soil characteristics over the entire slope by collecting fairly representative sample and determining the input soil parameter in the laboratory.

Figure 1. Typical view of slope failure near Gambhar Bridge on NH 21

The slope stability was assessed using the deterministic analysis and commonly used methods of analysis along with the software SLOPEW and PLAXIS (including dynamic loading input). Finally, the results obtained from the two approaches are compared and their efficacy for slope stability is determined. The site selected for the study is located in district Bilaspur, Himachal Pradesh, India on NH-21 highway namely Gambhar bridge. The height of the site is 1230 meter above sea level respectively. The study area lies in earthquake zone IV at latitude 31° 20´ N and longitudes 76º 45’ E. Average annual rainfall of the area is around 135 cm. A typical view of the slope failure is shown in figure 1.

Determination of basic geometrical characteristics of the slope was done using total station survey. Total station surveying was done for both the sites in order to generate contour maps of the slopes. The reduced levels, horizontal distance, vertical and horizontal angle readings were recorded using total station. These are fed as input in the software LISCAD to generate contour map as shown in figure 2. Three predominant sections 1-1, 2-2 and 3-3 of slope failure have been identified on the basis of the field observations as indicated in figure 3.

Fairly large numbers of representative samples of soil were collected from soil slope considering the variability of soil strata throughout the extent of slope. The soil parameters for the drained conditions were determined. The mean value of different properties was calculated. Typical results obtained for different properties are summarized in table 1.

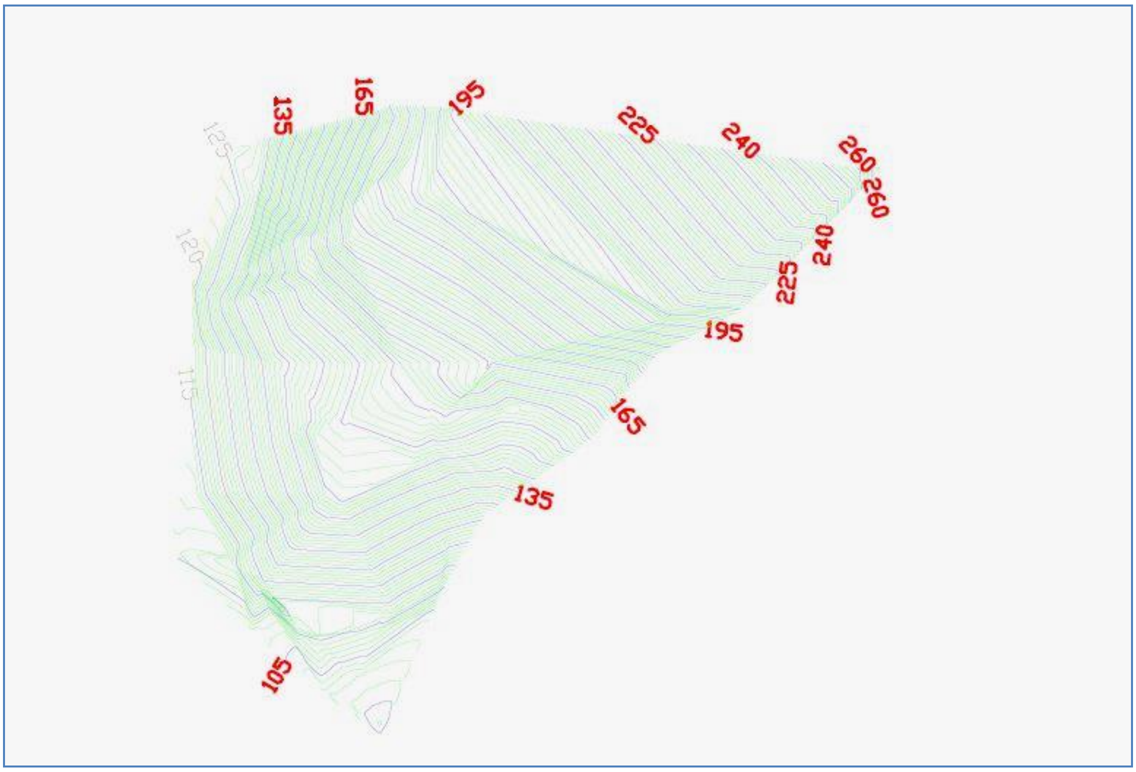

Figure 2. Contour map of site (near Gambhar bridge)

Table 1. Properties of different soil samples

S. No.	Water content (%)	Density (kN/m³)	Cohesion (kN/m²) (IS 2720 Part XIII, 1972)	Angle of internal friction (Φ) (IS 2720 Part XIII)[24]
1	6.80	17.63	27.16	9.85
2	3.69	18.97	19.03	24.96
3	14.79	17.39	7.57	21.4
4	14.85	19.79	18.24	15.97
5	13.95	22.40	5.13	21.03
6	12.56	20.06	17.72	21.76

Results and Analysis

Deterministic Approach

The traditional methods of slope stability normally use single valued parameters to analyze the characteristics of a slope. The output from traditional analysis methods yields single valued estimates of factor of safety of the stability of a slope. However, the parameters governing the stability of a slope vary considerably throughout the extent of the slope. Most commonly employed method of analysis of the stability a slope is Bishop's method [16] which yields the factor of safety as:

$$F = \frac{1}{\sum W \sin\alpha} \sum [cb + \tan\emptyset (W - Ub)] \frac{\sec\alpha}{1 + \frac{\tan\emptyset \tan\alpha}{F}} \qquad (1)$$

Where, F = Factor of safety, W = weight of slice, c = cohesion, b = width of slice, α = angle of inclination of slope, ∅ = angle of internal friction and U = pore pressure at each slice.

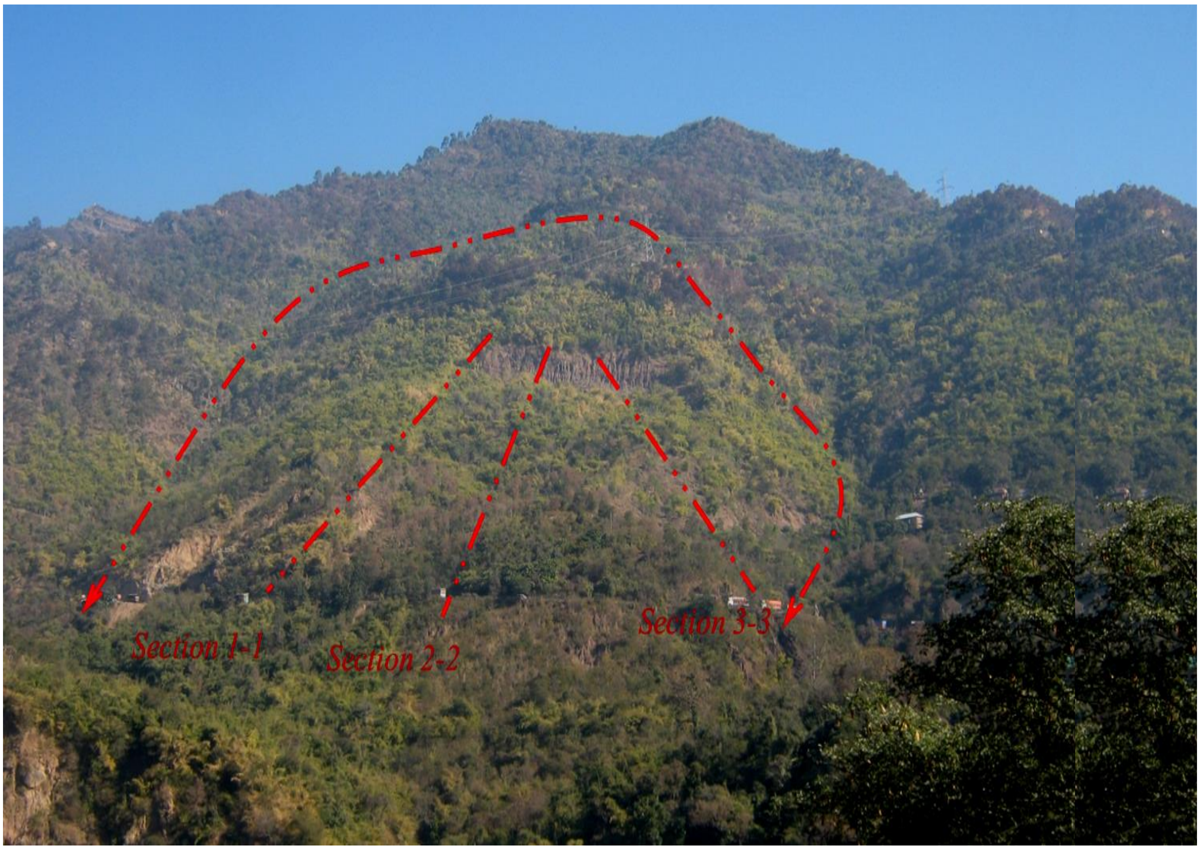

Figure 3. Three sections selected for slope stability analysis

An iterative analysis is necessary to obtain the factor of safety. Since this is a trial and error method, the assumed factor of safety F is entered with respect to which the new factor of safety is calculated and the iteration process is continued till the difference between the two values of factor of safety calculated is negligible. Three different sections namely 1-1, 2-2 and 3-3 were analyzed using SLOPE-W module of Geo Studio. The factor of safety for different sections was calculated with the help of different deterministic method namely ordinary method, Bishop's method [16], Janbu [17] method and Morgenstern Price Method [18][19]. Table 2 shows the values of factor of safety with the help of different methods. The results indicate that the slope is critically stable at sections 1-1 and 2-2 but the slope is unstable at section 3-3. The results show that the factor of safety values given by ordinary method of slices and Janbu method are in close proximity whereas the values indicated by Bishop's method and Morgenstern Price method are closer. However, the factor of safety determined using all methods for section 3-3 is nearly same which indicates that the factor of safety values is dependent upon slope geometry and characteristics.

Table 2. Factor of safety calculated for different sections using deterministic analysis

Sections	Ordinary Method	Bishop Method	Janbu Method	Morgenstern Price Method
1-1	1.041	1.071	1.039	1.069
2-2	1.091	1.245	1.086	1.128
3-3	0.839	0.840	0.818	0.839

Further, the slope sections have been analyzed as infinite slope using a MATLAB program. A MATLAB code was written for the slope stability considering the slope as infinite slope. The results obtained from code are represented through table 3. The results show that for an infinite slope the factor of safety values are very low even under dry condition and particularly very low under the condition when the tension crack is filled with water. The results, however, are not observed to be realistic as the slope is a finite one.

Table 3. Factor of safety for infinite slope

Section	Dry condition	Tension crack filled with water
1-1	0.90	0.44
2-2	0.88	0.43
3-3	0.78	0.38

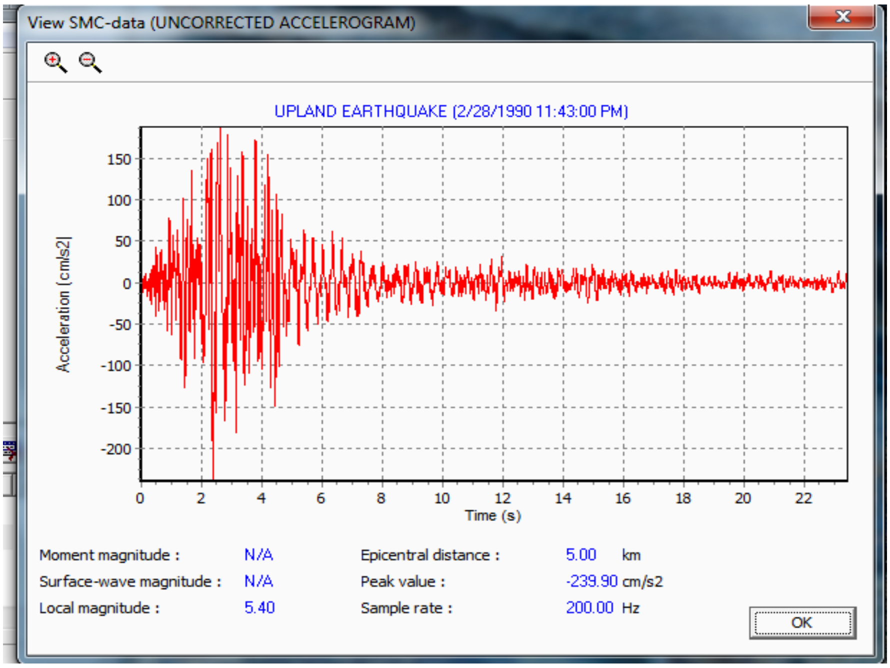

Figure 4. Accelerogram used to simulate dynamic loading input

PLAXIS version 8 has been used to carry out two-dimensional finite element analysis. A Plane strain model is used for geometries with a (more or less) uniform cross section and corresponding stress state and loading scheme over a certain length perpendicular to the cross section (z-direction). Displacements and strains in z-direction are assumed to be zero. However, normal stresses in z-direction are fully taken into account. In this software after defining geometry of the problem, assigning geotechnical specifications of soil layers, segment material and water table, settlement calculation and stress-strain analysis are done through two phases by stage construction capability of the software. The 15-node triangle is the default element which provides a fourth order interpolation for displacements and the numerical integration involving twelve Gauss points (stress points) has been used. The 15-node triangle is a very accurate element that has produced high quality stress results for difficult problems, as for example in collapse calculations for incompressible soils. Three different sections have been analyzed with the help of PLAXIS 8.2 for the following four different conditions: (i) slope is dry, (ii) tension crack filled with water, (iii) cohesion reduced to zero due to vibrations and (iv) Dynamic loading input. The accelerogram used to simulate the dynamic loading input used in the analysis is shown in figure 4.

The finite element modeling of the most critical failure plane at section 1-1 with simulation of dynamic loading is shown in figure 5. The deformed mesh at section 1-1 with simulation of dynamic loading at most critical plane is shown in figure 6.

The boundary elements, particularly at the sharp transitions are observed to incur appreciable displacements. The elements at the toe of the slope indicate large displacements and lead to stress concentrations as is observed from figure 7 showing the stress distribution across the cross-section 1-1. Similarly, the finite element modeling of the most critical failure plane along with the deformed mesh and the stress distribution at sections 2-2 and 3-3 for other conditions was performed to determine factor of safety. The factor of safety values computed using PLAXIS incorporate the consideration of all soil parameters and include the effect of tension crack filled with water, loss of soil cohesion due to vibrations as well as the effect of dynamic loading. The results obtained from PLAXIS for four different conditions are given in table 4.

Table 4. Factor of safety using PLAXIS

Section	1-1				2-2				3-3			
Case	I	II	III	IV	I	II	III	IV	I	II	III	IV
Factor of safety	1.117	1.061	0.983	0.890	1.072	0.886	0.743	0.652	0.934	0.927	0.549	0.456

The factor of safety values indicate that the slope is critically stable at section 1-1 under the two conditions for dry slope and when the tension crack filled with water; whereas at section 2-2 for dry condition of slope only. For the remaining conditions i.e. when cohesion is reduced to zero due to vibrations and under dynamic loading, the slope is unstable at section 1-1 and for the section 2-2 the slope is unstable for the remaining three conditions. At section 3-3, the slope is unstable for all the loading conditions which indicate that the slope stabilization measures have to be undertaken at this section.

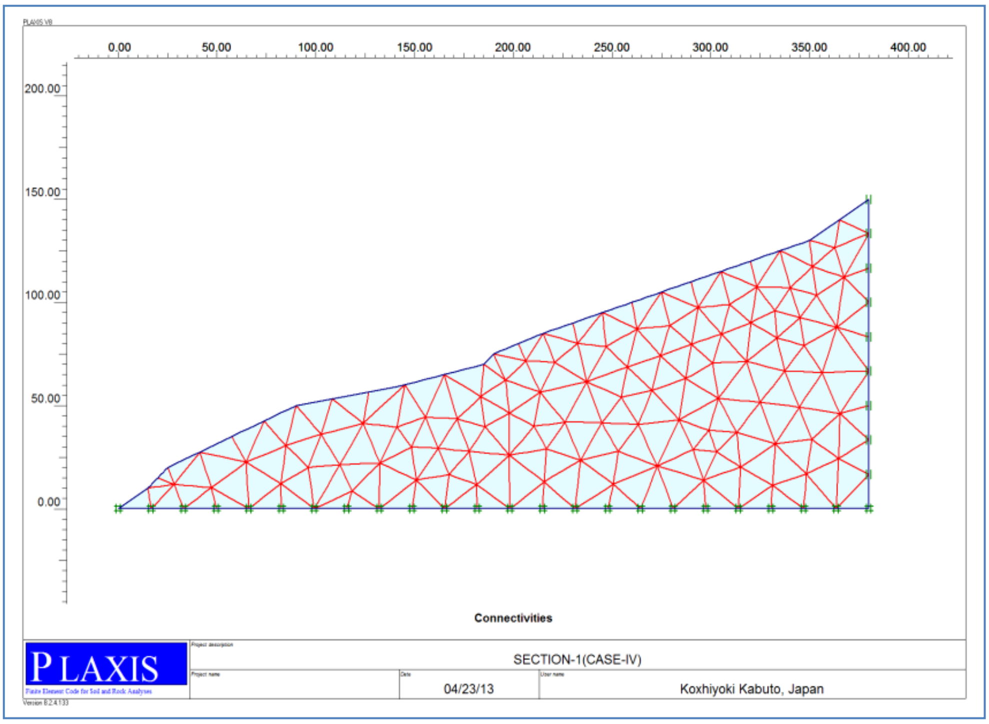

Figure 5. Finite element modeling at section 1-1 with dynamic loading at most critical plane

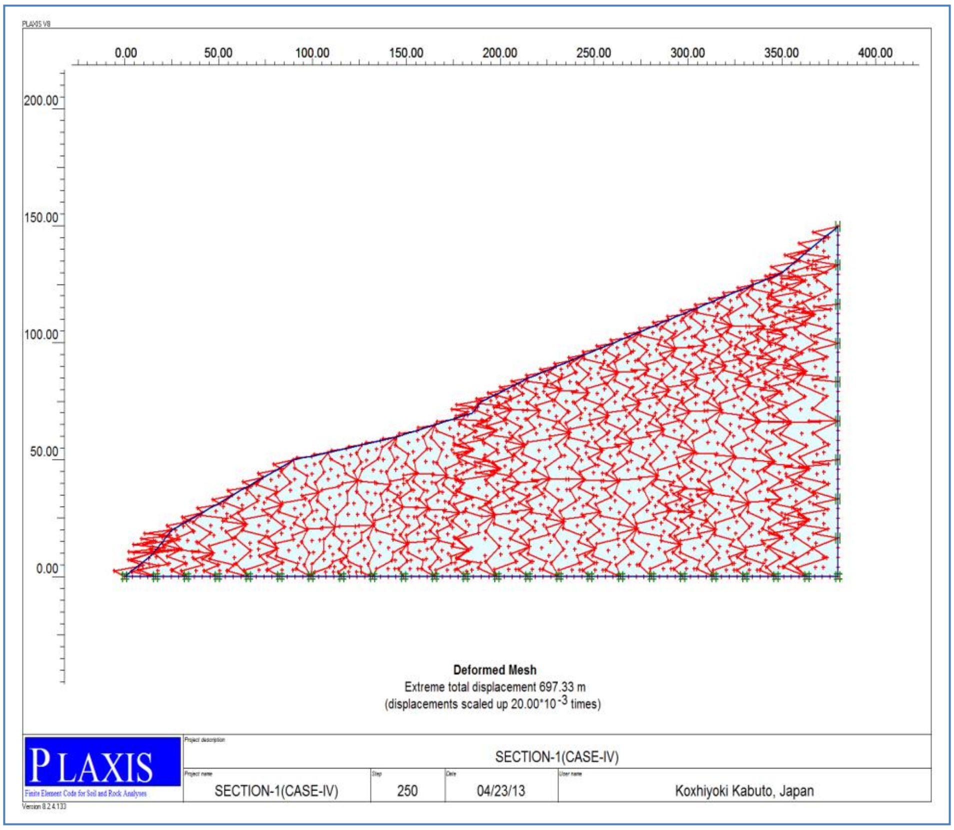

Figure 6. Deformed mesh at section 1-1 with dynamic loading at most critical plane

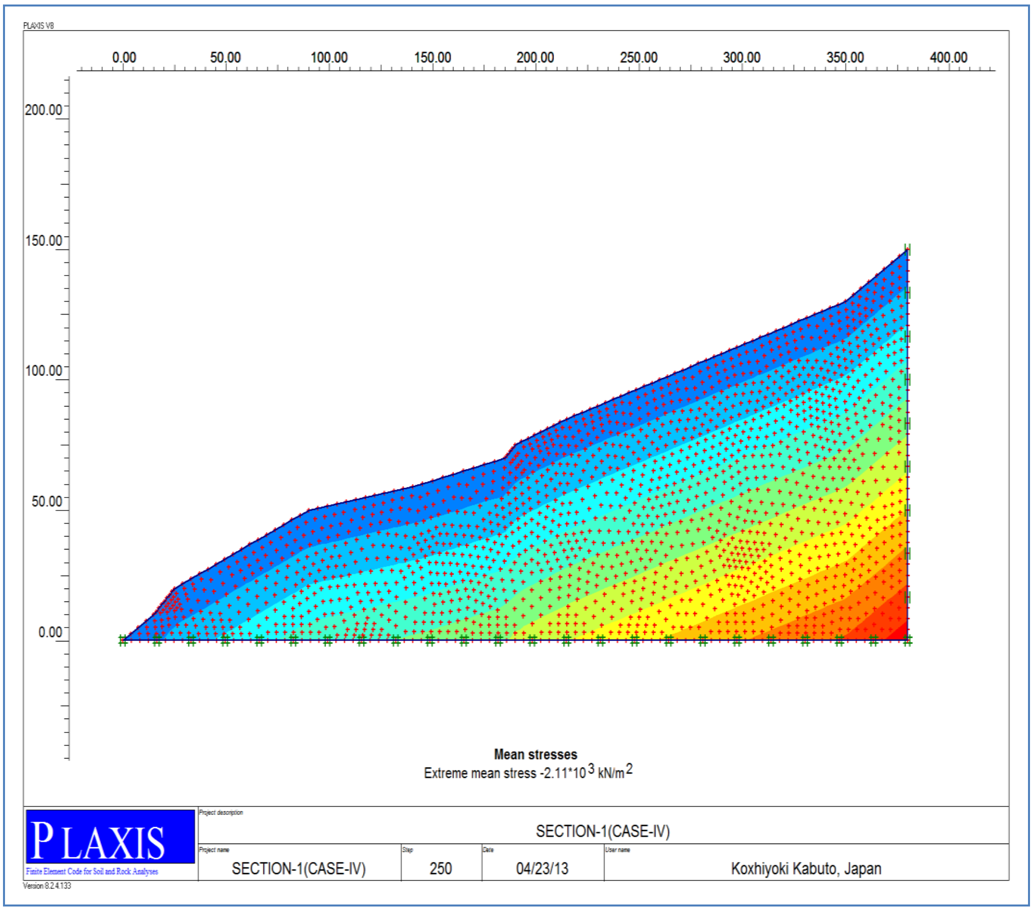

Figure 7. Stress distribution at section 1 with dynamic loading at most critical plane

Probabilistic Slope Stability Analysis Methods

Slope stability is one of the most important issues of concern to geotechnical engineers. Analysis of slope stability is composed of many uncertainties pertinent to lack of accurate geotechnical parameters, inherent spatial variability of geo-properties, change of environmental conditions, unpredictable mechanisms of failure, simplifications and approximations used in geotechnical models. Due to the importance of dam projects and its pertinent costs, determination of dam performance has a significant consequence to decision makers. With respect to the uncertainties of geotechnical parameters, utilizing risk analysis is inevitable in dam projects [20]. Conventional approaches do not take into account many uncertainties in their calculations quantitatively. Also, several conservative safety factors are using to cover some uncertainties which in most cases are more than required, and in some cases less than what is necessary. Actually, it is not possible to distinguish the accurate effect of these safety factors on safety level. By contrast, in probabilistic approaches the safety determination applies more accurately and clearly [21]. Uncertainties in soil properties, environmental conditions, and theoretical models are the reason for a lack of confidence in deterministic analyses [22]. Compared to a deterministic analysis, probabilistic analysis takes into consideration the inherent variability and uncertainties in the analysis parameters. Judgments are quantified within a probabilistic analysis by producing a distribution of outcomes rather than a single fixed value. Thus, a probabilistic analysis produces a direct estimate of the distribution of either the factor of safety or critical height associated with a design or analysis situation. There are several probabilistic techniques that can be used to evaluate geotechnical situations. Specifically, for geotechnical analysis, researchers have conducted probabilistic evaluations using Monte Carlo simulations, Point Estimate method, and in conjunction with a probabilistic analysis a reliability assessment. Monte Carlo probabilistic analysis has been performed in this study.

Monte Carlo Simulation

The Monte Carlo method was developed in 1949 by John von Neumann and Stanislaw Ulam [23]-[25]. They designated the use of random sampling procedures for treating deterministic mathematical situations. The foundation of the Monte Carlo gained significance with the development of computers to automate the laborious calculation. The first step of a Monte Carlo simulation is to identify a deterministic model where multiple input variables are used to estimate a single value outcome. Step two requires that all variables or parameters be identified. Next, the probability distribution for each independent variable is established for the simulation model, (i.e., normal, beta, lognormal, etc.). Next, a random trial process is initiated to establish probability distribution function for the deterministic situation being modeled. During each pass, a random value from the distribution function for each parameter is selected and entered into the calculation.

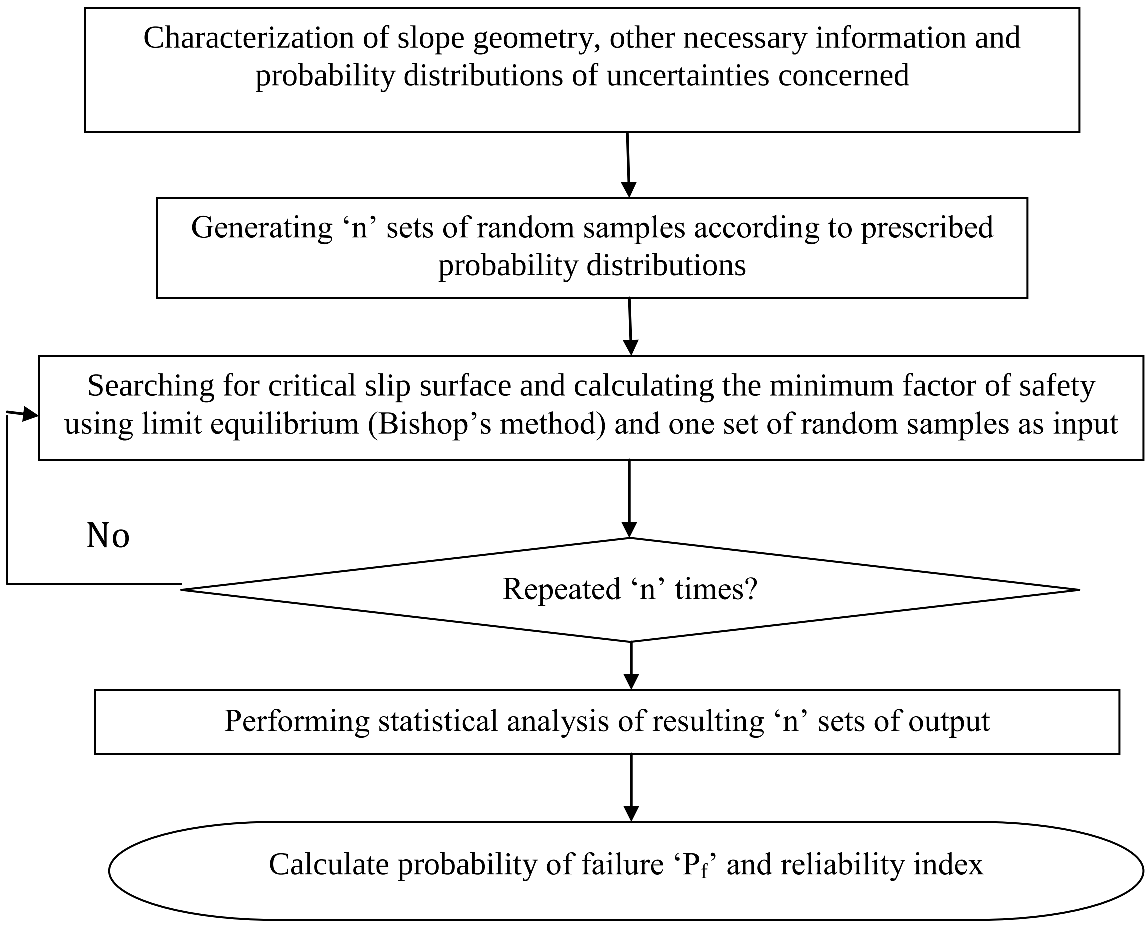

Figure 8. Steps involved in Monte Carlo simulation

Numerous solutions are obtained by making multiple passes through the program to obtain a solution for each pass. The appropriate number of passes for an analysis is a function of the number of input parameters, the complexity of the modeled situation, and the desired precision of the output. The final result of a Monte Carlo simulation is a probability distribution of the output parameter. Monte Carlo simulation is a powerful tool for slope stability risk analysis. An iterative process using deterministic methods of slope stability analysis is applied in this technique. Monte Carlo simulation is a popular method of slope stability risk analysis among engineers because of its simplicity and no need of comprehensive mathematical and statistical knowledge. This method consists of four steps (figure 8) as below [26][27]: (a) choosing a random value for each input variable according to assigned probability density function, (b) calculating factor of safety by using a proper deterministic slope stability analysis method (such as Janbu, Bishop, Spencer, etc.)[16][17][28] based on selected values in step 1, (c) repeating steps 1 and 2 for many times as necessary and (d) determining distribution function of factors of safety and probability of failure. For the above mentioned sections, probabilistic analysis was performed using Monte Carlo simulations. According to Monte Carlo simulation method, a random value has been selected for each input parameter based on the assigned probability density function and its amplitude. Theoretically, more are Monte Carlo trials the more accurate the solution will be, but the number of required Monte Carlo trials is dependent on the level of confidence in the solution and the amount of variables being considered. Statistically, the following equation has been recommended [29]:

$$N = \left(\frac{d^2}{4(1-e)^2}\right)^\wedge m \qquad (1)$$

Where: N = number of Monte Carlo trials, d = the normal standard deviation corresponding to the level of confidence, e = desired level of confidence, and m = number of variables. The probability density functions of unit weight, cohesion and angle of internal friction, φ adopted in the analysis are shown in figures 9, 10 and 11 respectively. Based on equation (1) for three variables (unit weight, cohesion and phi) and for 90% confidence level 309610 trials have been done with respect to standard deviation of 1.645. The various variables involved in the study, their mean values and type of distribution adopted is summarized in table 5. Reliability index is a rational probabilistic criterion for safety level which can be calculated by the following equation:

$$\beta = (E(FS) - 1)/\sigma(FS) \qquad (2)$$

Table 5. Variables involved in Monte Carlo simulations in this study

Variable	Mean value	Standard deviation	Distribution adopted
Unit weight	19.37	1.84	Normal
Cohesion	15.81	8.13	Normal
Phi	19.16º	5.40	Normal

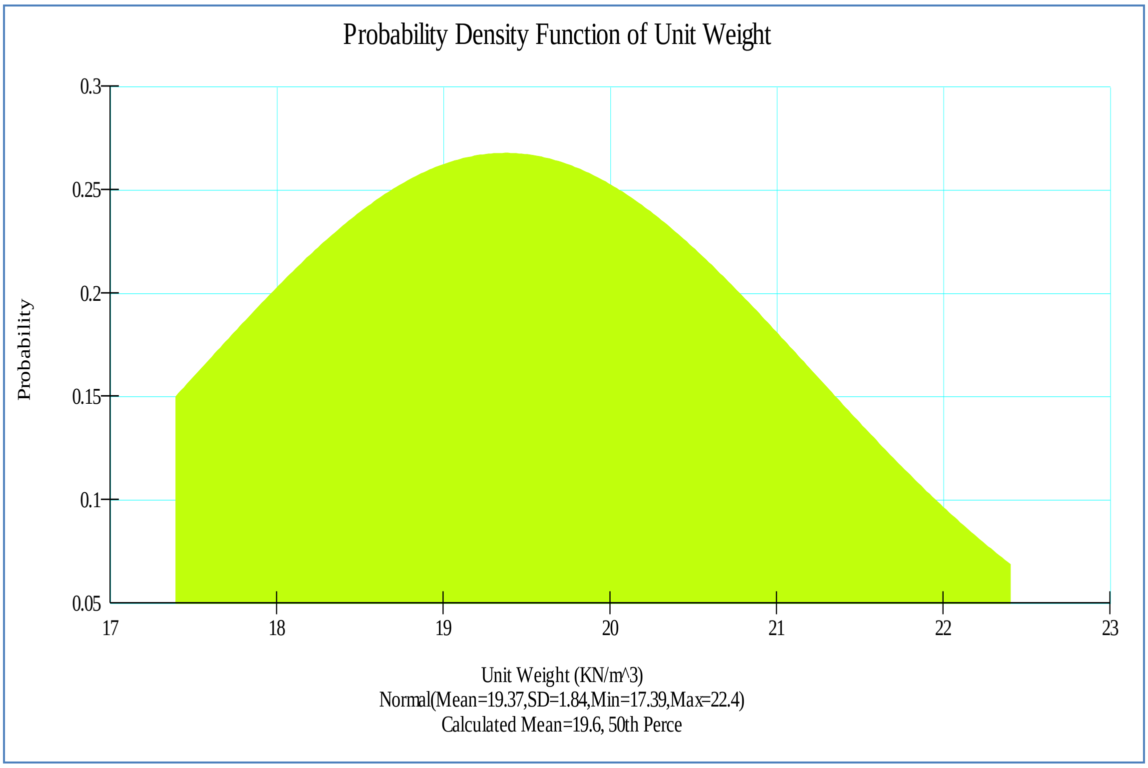

Figure 9. Probability density function of unit weight

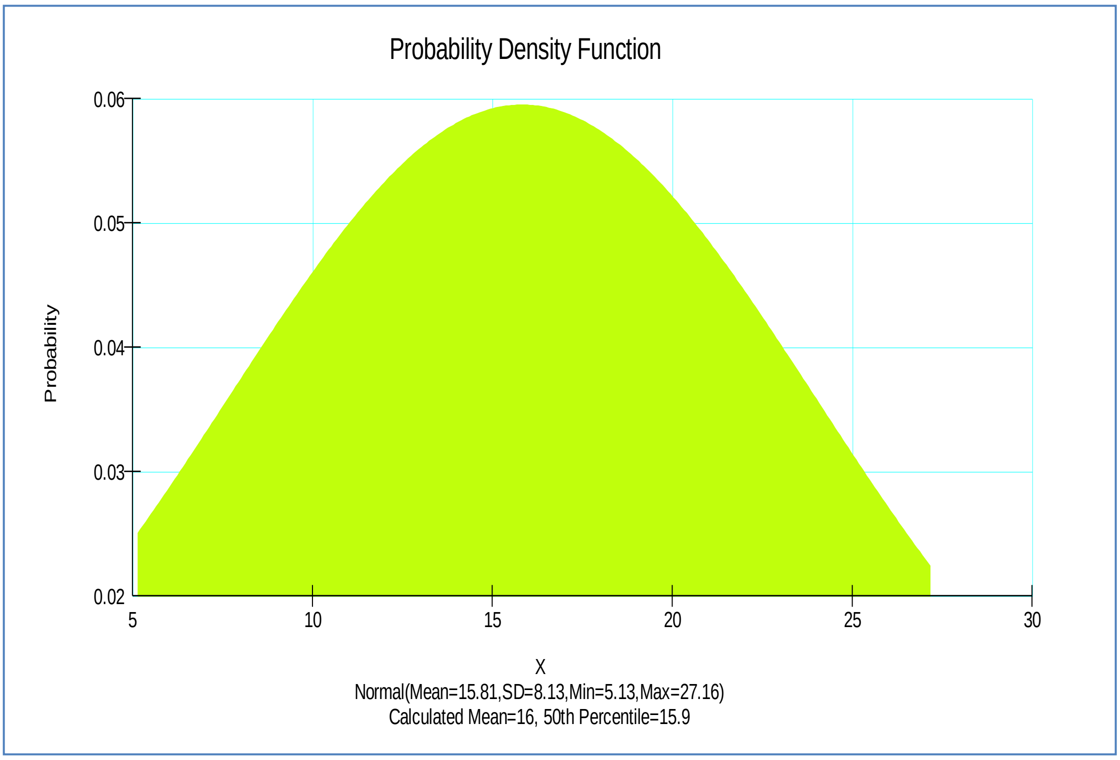

Figure 10. Probability density function of cohesion

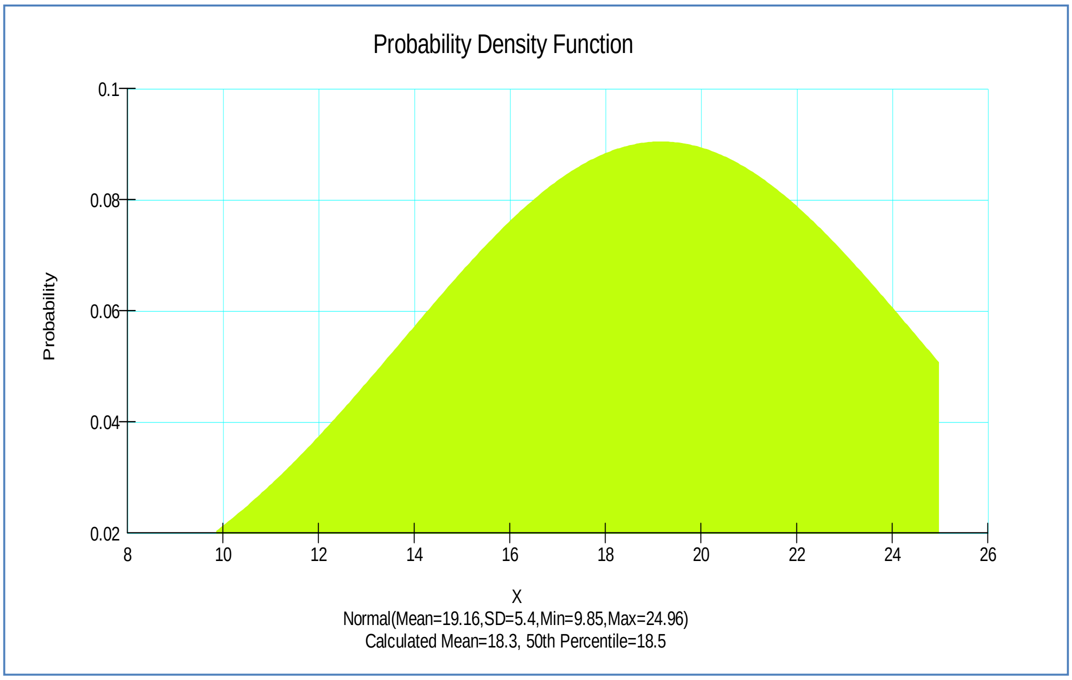

Figure 11. Probability density function for Phi, φ (angle of internal friction)

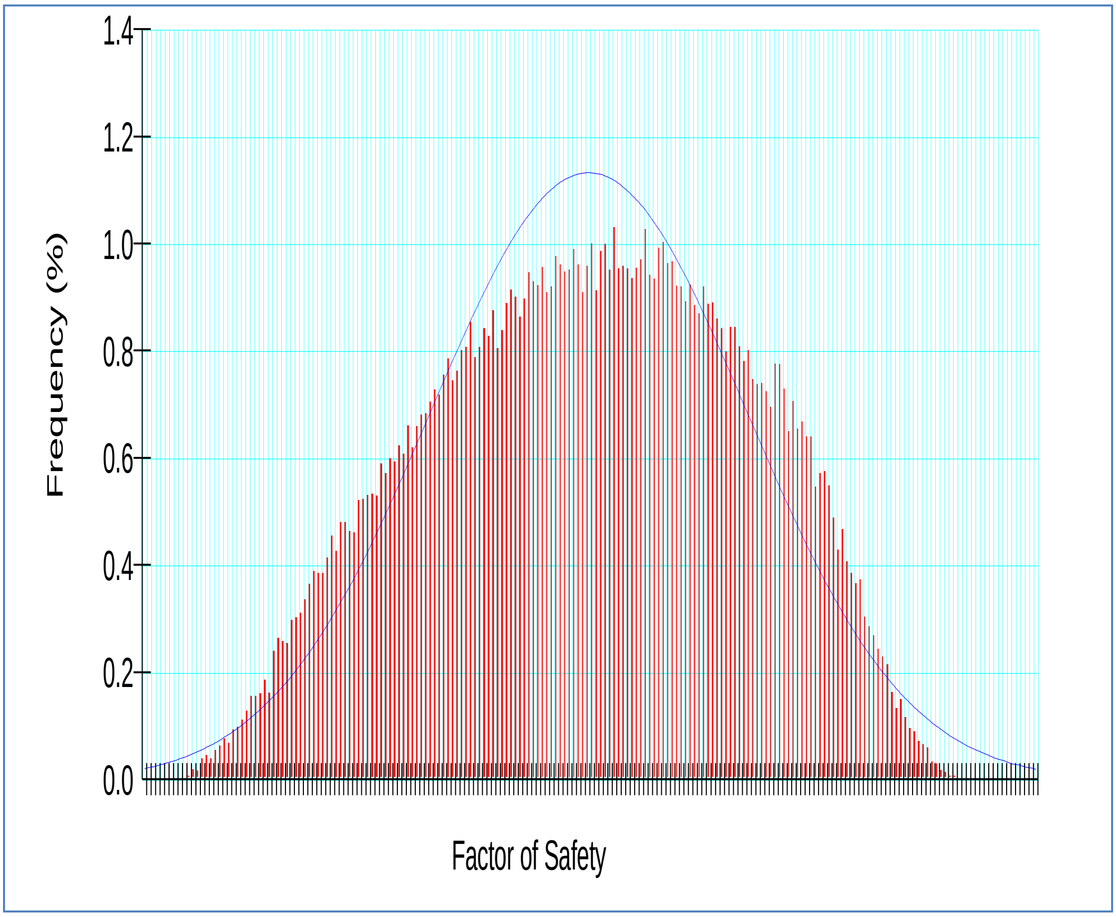

Figure 12. Probability distribution for factor of safety at section 1-1

Where E(FS) and $\sigma(FS)$ are average and standard deviation of safety factors respectively. Reliability index represents the level of reliability of an engineering system and reflects the effects of uncertain parameters on probabilistic analysis. The probability distribution for factor of safety at section 1 - 1, section 2 -2 and section 3 - 3 are shown in figures 12, 13 and 14. The results of probabilistic analysis are represented in Table. 6. As it appears from the table 6 that section 3-3 is most vulnerable towards failure. According to U.S. Army Corps of Engineers [20], for embankment dams, slopes with reliability index of more than 3 are stable. But from table 6, it can be observed that all three sections are having reliability index less than 3 so this slope is not reliable and requires slope stabilization techniques to stabilize it.

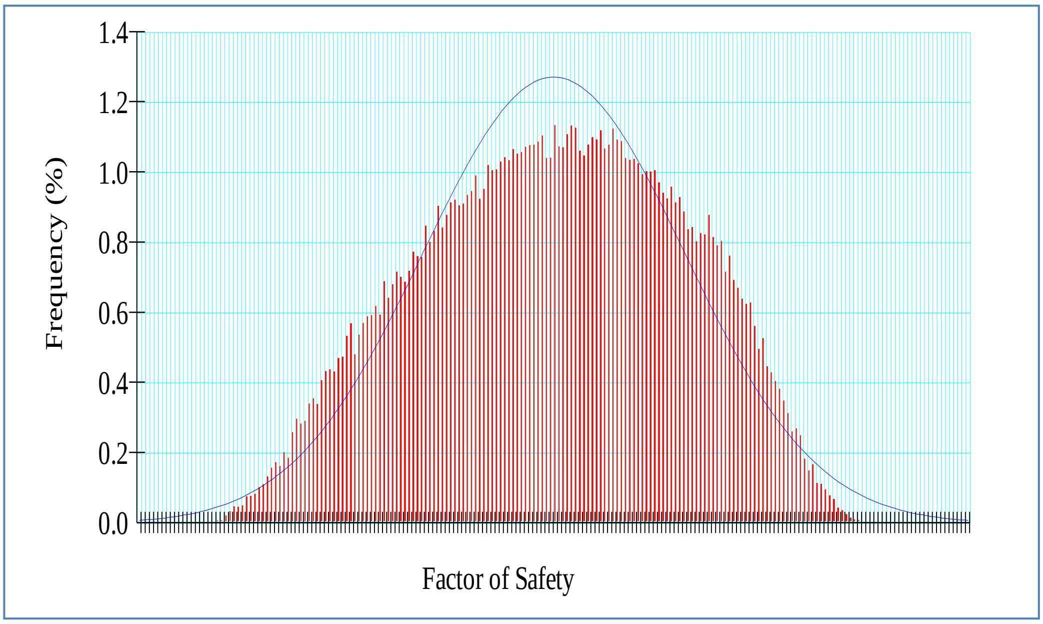

Figure 13. Probability distribution for factor of safety at section 2-2

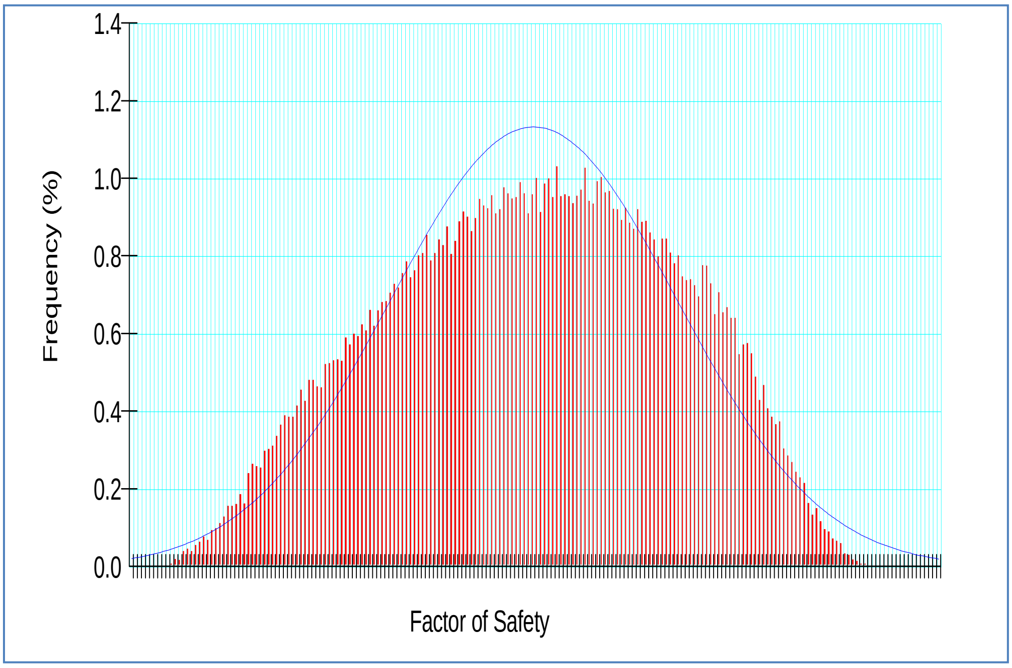

Figure 14. Probability distribution for factor of safety at section 3-3

The results of probabilistic analysis infer that, corresponding to the maximum factor of safety values, the slope at section 2-2 is stable but critically stable at sections 1-1 and 3-3. However, the minimum values of factor of safety indicate that the slope is highly unstable at all the three sections. Corresponding to mean value of factor of safety, slope is critically stable at sections 1-1 and 2-2 but unstable at section 3-3 (higher probability of failure). The results of probabilistic analysis are observed to be more realistic as compared to the results obtained from other methods. Further, the results obtained from probabilistic analysis can be used to determine the probability of failure corresponding to a particular of factor of safety. Therefore an allowable risk criterion can be used to establish a consistent target for the design process. The reliability of the proposed factor of safety can be assessed and the design of the cut slope can be decided accordingly.

Table 6. Results of probabilistic analysis

Section	Mean factor of safety	Min. factor of safety	Max. factor of safety	Reliability index	Probability of failure	Standard deviation
1-1	1.025	0.52	1.48	0.125	44.77	0.205
2-2	1.083	0.54	1.58	0.394	35.74	0.212
3-3	0.8059	0.40	1.17	-1.234	87.78	0.157

Conclusions

The deterministic approach considering different methods of stability analysis namely ordinary method, Bishop’s method, Janbu's method and Morgenstern Price method using the iterative capabilities of software SLOPEW and PLAXIS (using dynamic loading input) have been used to assess the stability of a large natural slope. Deterministic approach generally yields conservative values of factor of safety since the input parameters assigned are single valued and the spatial variation of the input parameters is not accounted for. The results obtained from probabilistic approach can be used to determine the probability of failure corresponding to a particular of factor of safety and an allowable risk criterion can be used to establish a consistent target for the design process. The factor of safety obtained from the deterministic analysis indicates that Janbu's method gives the least factor of safety and Bishop's method giving the highest one with Morgenstern Price method yielding the values closer to Bishop's method. While considering the slope as an infinite slope, a smaller factor of safety was obtained which appears to be unrealistic. From probabilistic analysis, it is observed that section 3-3 is most vulnerable towards failure with reliability index of -1.234. Section 1-1 and section 2-2 too have reliability index less than 3 (recommended one for a slope for its stability). Thus, whole slope is vulnerable towards failure and that can be seen during rainy season when the slope faces failures and leads to disruption of traffic on the national highway. Further, the slope is vulnerable towards the dynamic loading with factor of safety reduced to nearly 0.5 under the dynamic loading input.

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