Acoustic scattering by multiple spheroids using collocation multipole method

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Abstract

This paper presents a semi-analytical approach to solve the three-dimensional acoustic scattering problems with multiple spheroids subjected to a plane sound wave. The results can provide benchmark solutions that are useful for evaluating the accuracy of various numerical methods. To satisfy the Helmholtz equation in the spheroidal coordinate system, the scattered acoustic field is formulated in terms of radial and angular prolate spheroidal wave functions which also satisfy the radiation condition at infinity. The multipole method, the directional derivative and the collocation technique are combined to propose a collocation multipole method in which the acoustic field and its normal derivative with respect to the non-local spheroidal coordinate system can be calculated without any truncated error, frequently induced by using the addition theorem for a multiply-connected domain problem. The boundary conditions are satisfied by collocating points on the surface of each spheroid. By truncating the higher order terms of the multipole expansion, a finite linear algebraic system is acquired. The scattered field can then be determined according to the given incident sound wave. The convergence analyses considering the specified error, the separation of spheroids and the wave number of an incident wave are first carried out to provide guide lines for the proposed method. Then the proposed results for acoustic scattering by one, two and three spheroids are validated by using the available analytical method and numerical methods such as boundary element method. Finally, the effects of the convexity and orientation of spheroid, the separation between spheroids and the incident wave number and angle on the near-field acoustic pressure and the far-field scattering pattern are investigated.

Keywords: Collocation multipole method, Acoustic scattering, Prolate spheroid, Helmholtz equation, Radial and angular prolate spheroidal wave function, Near-field acoustic pressure, Far-filed scattering pattern

Introduction

The subject of acoustic scattering has long attracted the attention of researchers in academia or industry because the results of corresponding studies can be found in many applications (Ingard, 2008) such as locating sound sources, noise control, etc. Although numerical methods such as finite element methods (FEM) and boundary element methods (BEM) can solve these problems, analytical solutions, if available, usually result in accurate and fast-rate convergence methodologies and provide physical insight into the problem under consideration. Furthermore, its results can also provide benchmark solutions that are useful for evaluating the accuracy of various numerical methods. Consequently, a semi-analytical approach to the problem of a plane sound wave scattered by multiple spheroids is presented in this work. The multipole method for solving multiply-connected domain problems was firstly proposed by Zȧviška (1913). The addition theorem is often employed to transform the multipole expansion into one of the local coordinate systems to satisfy the specified boundary conditions. However, we need to face a difficult formulation due to the infinite series form of the addition theorem.

Problem statement

Consider *L* arbitrary oriented prolate spheroids subjected to an incident plane sound wave shown in Fig. 1. There are *L*+1 observer coordinate systems to describe the problem considered here: $Oxyz$ is a global Cartesian coordinate system and $O_j\xi_j\eta_j\phi_j$, $j = 1,\ldots, L$, is the *j*th local prolate spheroidal coordinate system attached one of the *L* prolate spheroids. The position of each of the origins O_j with respect to global Cartesian coordinate system is given by (x^j, y^j, z^j). The scattered field can be expressed as an infinite sum of multipoles at the center of each spheroid as follows:

$$p^{(sc)}(\mathbf{r};\ \xi_1,\eta_1,\phi_1,\xi_2,\cdots,\xi_L,\eta_L,\phi_L)$$
$$=\sum_{j=1}^{L}\left[\sum_{n=0}^{\infty}\sum_{m=0}^{n} a_{mn}^{j}\, R_{mn}^{(3)}(c,\xi_j)S_{mn}(c,\eta_j)\cos m\phi_j+\sum_{n=1}^{\infty}\sum_{m=1}^{n} b_{mn}^{j}\, R_{mn}^{(3)}(c,\xi_j)S_{mn}(c,\eta_j)\sin m\phi_j\right]. \quad (1)$$

where $S_{mn}(c,\eta)$ is angular prolate spheroidal wave functions of degree n and order m, $R_{mn}^{(3)}(c,\xi)$ is radial prolate spheroidal wave function of the third kind and the coefficients a_{mn} and b_{mn} are to be determined by the boundary conditions.

An unbounded acoustic medium with a sound-hard prolate spheroid of a=1.0 and b=0.7 subjected to an incident sound wave was first considered. Graphs of the pressure intensity on the surface of a sound-hard spheroid for $\theta=\pi/2$ at $\phi=\pi/2$ are shown in Fig.2 as a function of the wave number ka by using various numbers of terms M in the multipole expansion. For the dimensionless incident wave number $ka\le 8$, the convergence is achieved when taking the number of terms M=12. Comparisons of the predicted error of the pressure intensity on the surface of a sound-hard prolate spheroid for $\theta=\pi/2$ at $\phi=\pi/2$ are shown in Fig.2 by using a base 10 logarithmic scale for the error, where the solid line represents the present method, the dashed line the BEM+CHIEF and the dotted line the BEM. The absolute error $\left\| p\right| - \left| p_a \right\|$ is used in Fig. 3, where $|p_a|$ denotes the magnitude of pressure for the analytical method and $|p|$ for one of several methods mentioned above.

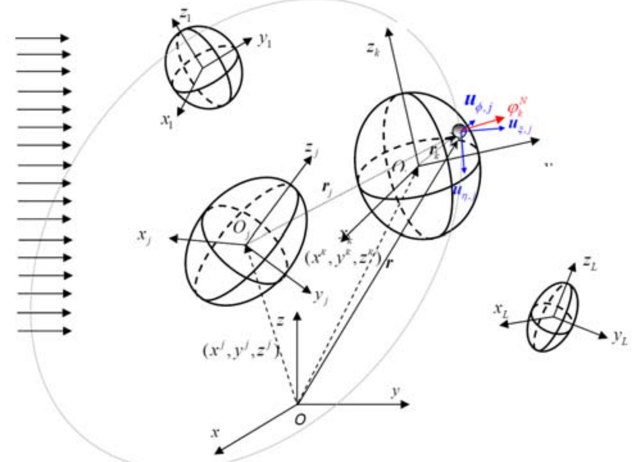

Fig.1 Problem statement for the acoustic scattering of a plane sound wave by multiple prolate spheroids and the associated coordinate systems

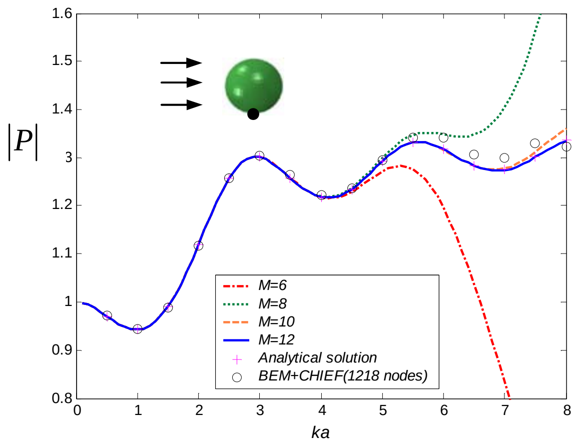

Fig. 2 Pressure intensity on the surface of a sound-hard spheroid for $\theta=\pi/2$ at $\phi=\pi/2$

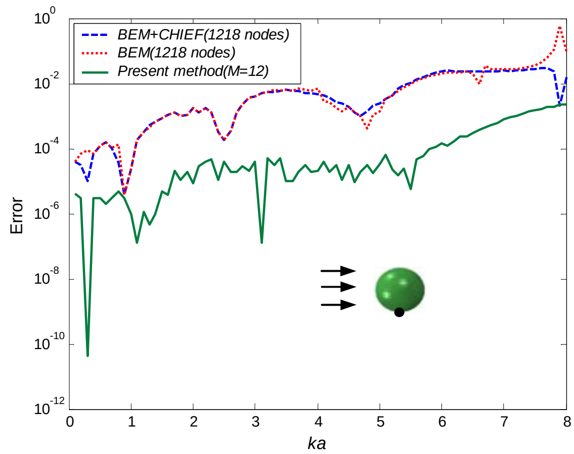

Fig.3 Comparisons of the predicted error of the pressure intensity

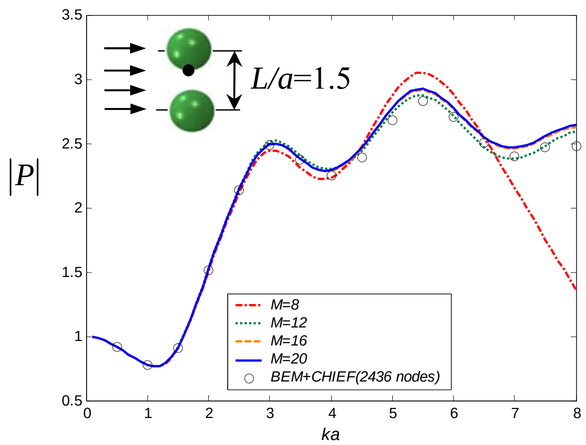

Fig.4 Pressure intensity on the surface of the superior spheroid for $\theta=\pi/2$ at $\phi=\pi/2$

For the case of two spheroids, graphs of the pressure intensity on the surface of the spheroid are shown in Fig. 4 as a function of the *ka* by using different numbers of terms *M* and the BEM. As the result for one sphere shown in Fig.2, it shows apparent deviation for the high wave number.

Conclusions

Three-dimensional acoustic wave scattered by multiple spheroids in an unbounded medium was semi-analytically solved by using the collocation multipole method. Instead of the addition theorem used by the traditional multipole method, the proposed non-local spheroidal derivative can exactly calculate the normal derivative with respect to different local spherical coordinates in the multiply-connected domain problem, free of truncation error caused by the addition theorem. The proposed algorithm has a concise formulation and is easily applicable to problems with canonical boundaries. It follows from numerical experiments that fictitious frequencies are not found in our formulation. As the separation is decreased, the near-field pressure intensity increases. On the other hand, the far-field scattering pattern becomes more fluctuated along the azimuthal angle as the separation is increased. It indicates that the effect of the space between scatterers on the near-field pressure intensity is opposite from that on the far-field scattering.

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