Random vibration analysis of structures with uncertain-but-bounded parameters

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Abstract

Random vibration response of structures with uncertain-but-bounded parameters under random process excitation is investigated in this paper. The interval natural frequencies, interval mean square displacements and stresses are analysed under the framework formed by the theories of structural dynamics and interval analysis. The lower and upper bounds of structural dynamic characteristics and random responses are determined by solving optimization problems. An improved particle swarm optimization algorithm, namely lower sequence initialized high-order nonlinear particle swarm optimization algorithm, is adopted to find their exact change ranges. Three numerical examples are provided to demonstrate the feasibility of the presented method. Quasi-Monte Carlo and Monte Carlo methods are also used to assess the effectiveness of the method.

Keywords: Interval analysis, random excitation, random vibration response, improved particle swarm optimization algorithm.

Introduction

Dynamic behavior of structures with uncertainty has been increasingly investigated as structural systems do have uncertainties in loads, geometric and material properties. The traditional technique to address these uncertainties is the probability theory. However, this technique is not available for most cases as there is not enough information to model probabilistic features. This situation requires other models of uncertainty which can be accessible for incomplete data. Taken interval analysis (Moore, 1966) into account, it can be employed to describe imprecision with known lower and upper bounds of altered features. The representation of interval analysis in structural engineering, first, was introduced to consider uncertainty in statics problems (Köylüoğlu et al., 1995). Many studies regarding interval analysis for static and dynamic problems of structures was developed by some authors (Moens and Vandepitte, 2001; Muhanna and Mullen, 2001; Qiu et al., 2005; Gao, 2007). The task of interval analysis is to achieve enclosures of system outputs which can be converted to optimization problems. Although there are various algorithms for treatment of optimization problems, PSO presented by Kennedy and Eberhart (1995) is more effective with fewer number of iteration to target the same or better results in comparison with others algorithms. (Hu et al., 2003; Elbeltagi et al., 2005). The effective application of PSO into different engineering problems including structural optimization has been recently shown and developed in the work of some authors. The improvement of PSO by means of using randomized low discrepancy sequences for initialized particles has been recently considered. Liu et al., (2013) employed the low-discrepancy sequences initialized high-order nonlinear particle swarm optimization algorithm (LHNPSO) for interval analysis of vehicle-bridge interaction system with simple model of structure. Meanwhile, there are still potential problems such as investigation of this method with multi-dimension particles for complex structures. In this paper, random vibration response of structures with uncertain-but-bounded parameters under random process excitation of space truss is studied with adoption of LHNPSO. This paper is organized as follows. The next section provides a brief introduction of interval dynamic analysis. Then, application of LHNPSO for interval analysis and the numerical example are presented in following sections. Conclusions are stated in the last section.

Interval dynamic analysis of structures under random process excitation

An interval variable $\mathbf{x^I}$ can also be denoted in as the following (Moore, 1966):

$$\mathbf{x^I} = \left[\underline{x}, \overline{x}\right] = x^c + \mathbf{\Delta x^I} \tag{1}$$

where $x^c = \left(\underline{x} + \overline{x}\right)/2$ and $\mathbf{\Delta x^I} = \left[-\Delta x, +\Delta x\right]$

Interval analysis is considered as design variables of structures are modeled as uncertainty- but-bounded values. Thus, structural parameters and responses are also interval values. The dynamic equation of structures under stationary process excitation can be expressed as the following:

$$\mathbf{M^I \ddot{u}^I}(t) + \mathbf{C^I \dot{u}^I}(t) + \mathbf{K^I u^I}(t) = \mathbf{-M^I}\{\mathbf{1}\}\ddot{a}(t) \tag{2}$$

where $\mathbf{M^I}$, $\mathbf{C^I}$ and $\mathbf{K^I}$ are the interval matrices with respect to mass, damping and stiffness of structures respectively. $\mathbf{u^I}(t)$, $\mathbf{\dot{u}^I}(t)$ and $\mathbf{\ddot{u}^I}(t)$ are interval vectors defining structural displacement, velocity and acceleration in that order. $\{\mathbf{1}\}$ is a column vector with all components 1 and $\ddot{a}(t)$ is the random ground acceleration. By employing Rayleigh's quotient in the form of modal analysis with normalization of modes and spectral matrices, frequency equation is expressed as:

$$\mathbf{W} = diag\left(\left(\omega^I\right)^2\right) = \left(\boldsymbol{\varphi}^\mathbf{I}\right)^T \mathbf{K^I} \boldsymbol{\varphi}^\mathbf{I} \tag{3}$$

where $\mathbf{W}$, $\boldsymbol{\varphi}^\mathbf{I}$ and $\mathbf{K^I}$ are the interval matrices of natural frequencies, normalized-natural modes and global matrices of stiffness.

The solution of coupled equations presented by Eq. (2) can be achieved by using Duhamel integral:

$$\mathbf{u^I}(t) = \int_0^t \boldsymbol{\varphi}^\mathbf{I} \mathbf{h^I}(t-\tau)\left(\boldsymbol{\varphi}^\mathbf{I}\right)^T \mathbf{M^I}\{\mathbf{1}\}\ddot{a}(t)\,d\tau \tag{4}$$

In this case, τ is assigned as 0. The interval matrix of response function is defined as:

$$\mathbf{h}^{\mathbf{I}}(t) = diag\left(h_j^I(t)\right)$$
$$h_j^I(t) = \begin{cases} \dfrac{1}{\omega_{jD}^I}\exp\left(-\xi_j\omega_{jD}^I\right)\sin\omega_{jD}^I t & t \geq 0 \\ 0 & t < 0 \end{cases} \quad (5)$$

By performing a Fourier transformation for the correlation function matrix and integrating Eq. (4) step by step, the mean square value matrix of the structural displacement $\left(\psi_{\mathrm{u}}^{\mathrm{I}}(\mathrm{t})\right)^2$ can be obtained as (Gao and Kessissoglou, 2007):

$$\left(\boldsymbol{\psi}_{\mathbf{u}}^{\mathbf{I}}(t)\right)^2 = \int_0^\infty \boldsymbol{\varphi}^{\mathbf{I}}\mathbf{H}^{\mathbf{I}}(\omega)\left(\boldsymbol{\varphi}^{\mathbf{I}}\right)^T \mathbf{M}^{\mathbf{I}}\{\mathbf{1}\}\{\mathbf{1}\}^T\left(\mathbf{M}^{\mathbf{I}}\right)^T \mathbf{S}_{\mathbf{p}}(\omega)\boldsymbol{\varphi}^{\mathbf{I}}\mathbf{H}_*^{\mathbf{I}}(\omega)\left(\boldsymbol{\varphi}^{\mathbf{I}}\right)^T d\omega \quad (6)$$

where $\mathbf{S}_{\mathbf{p}}(\omega) = diag\left(S_{pi}(\omega)\right)$ is the equivalent one-side power spectral density matrix of $\ddot{a}(t)$. In this paper, model of Kanai-Tajimi designated by (Lin and Yong, 1987) is employed to define random ground level accelerations as:

$$S_{pi}(\omega) = \frac{\left(1+4\left(\xi_g\omega/\omega_g\right)^2\right).S_0}{\left(1-\omega^2/\omega_g^2\right)^2+4\left(\xi_g\omega/\omega_g\right)^2} \quad (7)$$

The modal damping in this paper is given by $\xi_j = 0.01\left(j = 1,2,...,n\right)$. Parameters for the equivalent one-side power spectral density are defined as the following: $\omega_g = 16.5\left(rad/s\right), \xi_g = 0.7, S_0 = 15.6\left(cm^2/s^3\right)$.

$\mathbf{H}^{\mathbf{I}}(\omega)$ is the interval matrix with respect to the frequency response function matrix of the structures expressed as:

$$\mathbf{H}^{\mathbf{I}}(\omega) = diag\left(H_j^I(\omega)\right)$$
$$H_j^I(\omega) = \frac{1}{\left(\omega_j^I\right)^2-\omega^2+i2\xi_j\omega_j^I\omega}, \qquad j = 1,2,...,n \quad (8)$$

where $i = \sqrt{-1}$ is the complex number. $\mathbf{H}_*^{\mathbf{I}}(\omega)$ is interval matrix termed as complex conjugate matrix of $\mathbf{H}^{\mathbf{I}}(\omega)$.

The mean square value of the kth degree of freedom of structural displacement:

$$\left(\psi_{\mathrm{uk}}^{\mathrm{I}}(t)\right)^2 = \int_0^\infty \vec{\boldsymbol{\varphi}}_{\mathbf{k}}^{\mathbf{I}}\mathbf{H}^{\mathbf{I}}(\omega)\left(\boldsymbol{\varphi}^{\mathbf{I}}\right)^T \mathbf{M}^{\mathbf{I}}\{\mathbf{1}\}\{\mathbf{1}\}^T\left(\mathbf{M}^{\mathbf{I}}\right)^T \mathbf{S}_{\mathbf{p}}^{\mathbf{I}}(\omega)\boldsymbol{\varphi}^{\mathbf{I}}\mathbf{H}_*^{\mathbf{I}}(\omega)\left(\vec{\boldsymbol{\varphi}}_{\mathbf{k}}^{\mathbf{I}}\right)^T d\omega$$
$$k = 1,2,...,n \quad (9)$$

where $\vec{\boldsymbol{\varphi}}_{\mathbf{k}}^{\mathbf{I}}$ is the kth line vector of the matrix $\boldsymbol{\varphi}^{\mathbf{I}}$. Similarly, the interval variable of the eth element stress response of 3D trusses can be determined as:

$$\left(\psi_{\sigma\mathrm{e}}^{\mathrm{I}}(t)\right)^2 = E_e^I\mathbf{BT}\left(\boldsymbol{\psi}_{\mathbf{ue}}^{\mathbf{I}}(t)\right)^2\mathbf{T}^T\mathbf{B}^T E_e^I \quad (10)$$

where $\left(\boldsymbol{\psi}_{\mathbf{ue}}^{\mathbf{I}}(t)\right)^2$ is the interval matrix of mean square value of $\mathbf{u}_{\mathbf{e}}(t)$

Random vibration response of structures using the low-discrepancy sequences initialized high-order nonlinear particle swarm optimization algorithm

Random vibration response with uncertain-but-bounded parameters is difficult to attain because of its complexity. The methodology of interval analysis is to determine the band for structural response based on structural parameters varying within a fixed range. In other words, lower and upper bounds are the minimum and maximum values of system outputs respectively. Practically, interval analysis problems are converted to optimization problems. In this paper, an improved particle swarm optimization algorithm is employed to solve presented problems.

The traditional technique of particle swarm optimization, known as PSO algorithm was proposed by Kennedy and Eberhart (1995). This algorithm is initialized with a population of random solutions called particles. Each particle moves through the multidimensional design space corresponding to fitness problem to search its optimal position via adjusting its position and velocity simultaneously as the following expression:

$$v_{ik}(t+1) = w(t+1)v_{ik}(t) + c_1 r_1\left(x_{ik}^{Pb}(t+1) - x_{ik}(t)\right) + c_2 r_2\left(x_{ik}^{Gb}(t+1) - x_{ik}(t)\right) \tag{11}$$

$$x_{ik}(t+1) = x_{ik}(t) + v_{ik}(t+1) \tag{12}$$

where k denotes any individual of particle. x_{ik}^{Pb} and x_{ik}^{Gb} denote the local best ever position and global best position of each individual of particle at iteration t respectively. c_1 and c_2 are the coefficients indicating the degree of directing to the better positions of particles. c_1 and c_2 are specified as $0 < c_1 + c_2 < 4$, regularly $c_1 = c_2 = 2$ (Perez and Behdinan, 2007). r_1 and r_2 indicate random numbers ranging between 0 and 1.

In recent studies, initialization of particle swarms by low-discrepancy sequences have been considering to obtain the better results (Liu et al., 2013). For LHNPSO, the constant acceleration coefficients are assigned as $c_1 = c_2 = 2$ and the nonlinearly decreasing inertial weight is varied as the following:

$$w(t+1) = w_{\max} - \left(w_{\max} - w_{\min}\right)\left(\frac{k}{k_{\max}}\right)^{1/\pi^2} \tag{13}$$

This improved technique of PSO algorithm, known as LHNPSO shows efficiency in interval analysis. This technique is employed to capture sharp bounds for structural responses for complex structures in the work of this paper where $w_{\max} = 0.95$ and $w_{\min} = 0.5$ in this paper. The interval structural responses **R**(x,t) such as natural frequency, displacement and stress are used as the fitness functions in the LHNPSO and the lower and upper bounds of these functions are respectively the minimum and maximum values of investigated variables.

$$\begin{cases} \overline{R}(x,t) = \max(\mathbf{R}(x,t)) \\ \underline{R}(x,t) = \min(\mathbf{R}(x,t)) \end{cases} \tag{14}$$

Numerical Example

The performance of work is examined by a space truss with 108 members, shown in Figure 1, in which structural responses are investigated by different methods, namely, LHNPSO with 100 iterations, Monte Carlo Simulation and Quasi-Monte Carlo Simulation with 5000 iterations simultaneously. The comparison of results among methods is shown by Table 3, Table 4, and Table 5 while the convergence history of selected responses is depicted in Figure 2, Figure 3, and Figure 4 with lower and upper bounds. The varied variables for structural design are described as $\mathbf{X^I} = \left\{ A_1^I, A_2^I, A_3^I, E^I, \rho^I \right\}$ where their middle values are defined as the following:

Table 1. Middle values of varied parameters

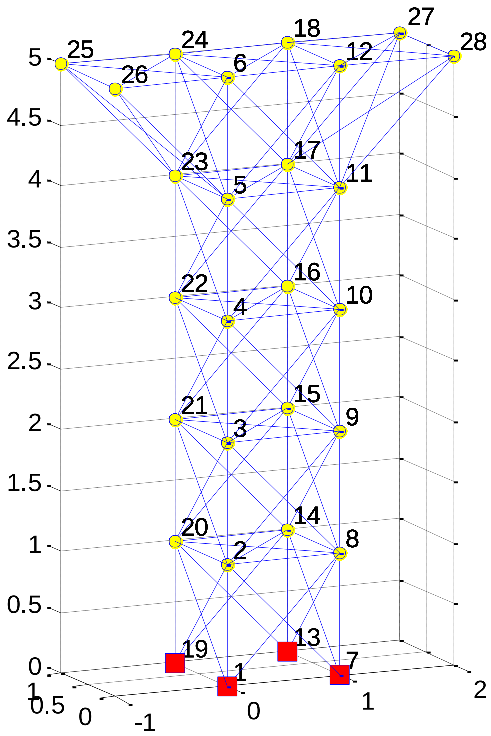
$A_1^c\left(m^2\right)$	$A_2^c\left(m^2\right)$	$A_3^c\left(m^2\right)$	$E^c\left(GPA\right)$	$\rho^c\left(kg/m^3\right)$
0.007	0.0084	0.014	200	7850

All varied parameters are restricted by the variance within $\alpha_1 = 10\%$. The structural members are divided into three groups as:

Table 2. Groups of structural members

Group	Area	Elastic Modulus	Density
1	A_1^I	E^I	ρ^I
2	A_2^I	E^I	ρ^I
3	A_3^I	E^I	ρ^I

Each group is defined by members as the following:



Group 1: 1-2, 2-3, 3-4, 4-5, 5-6, 7-8, 8-9, 9-10, 10-11, 11-12, 13-14, 14-15, 15-16, 16-17, 17-18, 19-20, 20-21, 21-22, 22-23, 23-24, 12-28, 18-27, 12-27, 18-28, 11-27, 17-28,11-28,17-27,27-28, 24-25, 6-26, 25-26, 24-26, 25-6, 26-5, 25-23, 26-23, 25-5.

Group 2: 14-20, 20-2, 2-8, 8-14, 2-14, 20-8, 15-21, 21-3, 3-9, 9-15, 15-3, 21-9, 16-22, 22-4, 4-10, 10-16, 16-4, 22-10, 17-23, 23-5, 5-11,11-17, 17-5, 23-11.

Group 3: Other members.

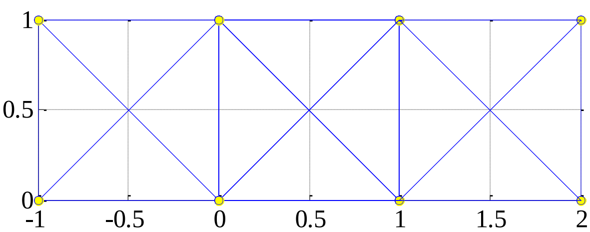


Figure 1. 3-D view and top view of the model

Table 3. Natural frequency

Natural frequency		
1st natural frequency $f_1(Hz)$		
Method	Upper bound	Lower bound
LHNPSO	34.027	26.294
QMCS	33.590	26.586
MCS	33.576	26.665
2nd natural frequency $f_2(Hz)$		
Method	Upper bound	Lower bound
LHNPSO	35.879	27.378
QMCS	35.224	27.963
MCS	35.182	27.791

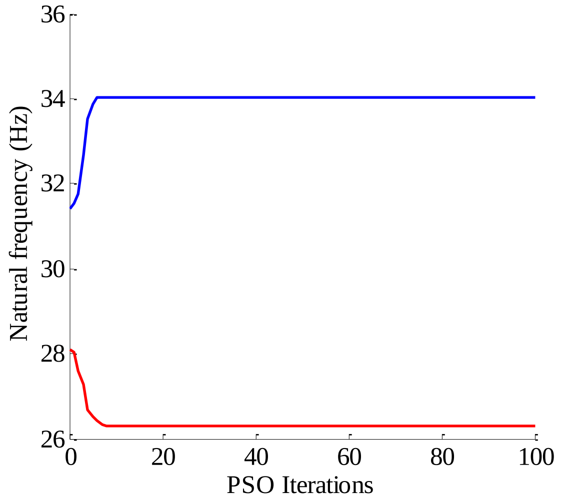


Figure 2. LHNPSO for $f_1(Hz)$

Table 4. Mean square root of displacement

Mean square root of displacement		
$\psi_{u12X}(mm)$		
Method	Upper bound	Lower bound
LHNPSO	14.080	8.054
QMCS	13.483	8.300
MCS	13.542	8.350
$\psi_{u27Y}(mm)$		
Method	Upper bound	Lower bound
LHNPSO	12.802	7.115
QMCS	12.141	7.347
MCS	12.501	7.336

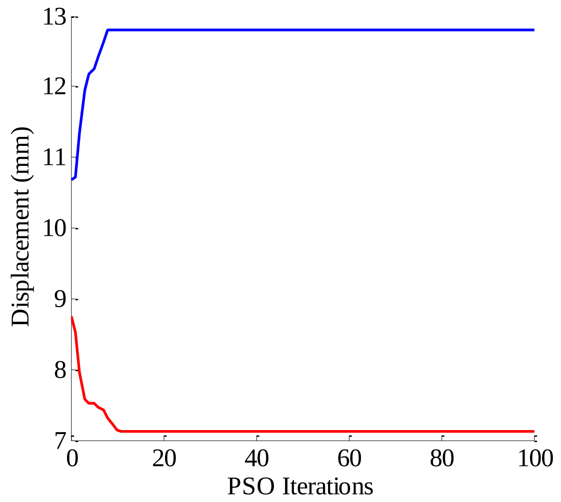


Figure 3. LHNPSO for $\psi_{u27Y}(mm)$

Table 5. Mean square root of structural stress

Mean square root of structural stress		
$\psi_{\sigma(5-6)}(MPA)$		
Method	Upper bound	Lower bound
LHNPSO	36.853	25.849
QMCS	36.124	26.366
MCS	36.053	26.330
$\psi_{\sigma(3-10)}(MPA)$		
Method	Upper bound	Lower bound
LHNPSO	29.057	20.079
QMCS	28.707	20.390
MCS	28.676	20.430

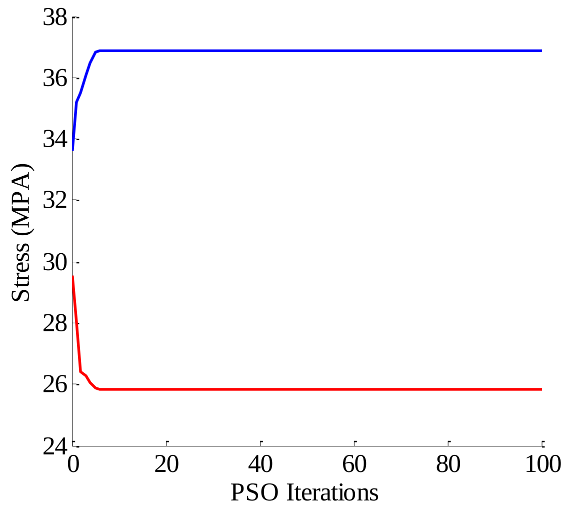


Figure 4. LHNPSO for $\psi_{\sigma(5-6)}(MPA)$

The selected responses are the first natural frequency, f_1, the second natural frequency, f_2, displacements at node 12 in direction X, ψ_{u12X}, and at node 27 in direction Y, ψ_{u27Y}, stresses of element 5-6, $\psi_{\sigma(5-6)}$, and element 3-10, $\psi_{\sigma(3-10)}$.

Conclusion

It is observed that bounded range of target solutions of LHNPSO embraces that of QMCS or MCS, as shown as Table 3, Table 4, Table 5. In other words, LHNPSO provide a sharp enclosure, namely close upper bound and close lower bound with reliable precision of convergence. In addition, it takes less time for LHNPSO to achieve the target solutions much more than those of QMCS or MCS, namely smaller iterations. It is shown that LHNPSO is suitable for dynamic analysis for structures.

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