Damage Identification of Beams based on Element Modal Strain Energy and Data Fusion with Reconstructed Modal Rotations

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Abstract

Damage identification of structures is always attractive to researchers because it plays an important role in the health monitoring in many civil engineering structures. When carrying out a health monitoring, sensors are usually laid on a beam to record acceleration signals, in which the modes of the beam can be extracted to construct indicators for detecting damages of the investigated beam. It should be noted that it is difficult to measure rotational signals of the beam at a position where sensors are laid, thus only the modal translations can be available. Although the pure modal translations can still be used to construct indicators and often it is the case, an indicator taking into account modal rotations is suggested in application to consider the effect of signal noise on the accuracy of measurement. In this paper, modal rotations were reconstructed by modal translations using the principle of static condensation. Then both modal translations and rotations were used to build an indicator based on an idea regarding element modal strain energy together with the theory of data fusion. The modal translations were extracted from accelerations recorded on a beam using stochastic subspace identification (SSI). Studies were carried out on choosing values of parameters in SSI in order to eliminate the effect of noise as nearly as possible. The simulation given by a FEM model and analyses of real accelerations recorded on a reinforced concrete beam show that this proposed damage indicator with elimination of noise effects is able to determine the locations of damage in the investigated beams.

Key Words: Damage identification, Element modal strain energy, Data fusion, Reconstructed modal rotations

Introduction

The initial defects of materials, improper construction methods and the combination of effects by long time load and environment, as well as sudden hazard may lead to damage in civil engineering structures. Structural health monitoring (SHM) can help to prevent significant damage and so to improve structural reliability and durability. Damage identification in early stage using recorded structural dynamic responses is an important branch of SHM. In practice, some dynamic responses are difficult to measure, such as rotations of a structure. So many damage identification methods utilize only structural translations, i.e. shear-type structural model (Hjelmstad et al, 1995). Damage indicators based on element modal strain energy were proved by researchers to be sensitive to damage and able to resist noise (Shi et al, 2002; Liu et al, 2004). Such indicators need structural rotational information and consequently researchers often used only simulations to verify them. In this paper, modal rotations were reconstructed by modal translations using the principle of static condensation (Guyan, 1965; Zhao and Li, 2003). Then both modal translations and rotations were used to construct a damage indicator based on element modal strain energy together with the theory of data fusion. The modal translations were extracted from accelerations recorded on a concrete beam using stochastic subspace identification (SSI). In order to eliminate the effect of noise as nearly as possible, studies were carried out on choosing values of parameters in SSI prior to constructing the indicator. The constructed damage indicator was applied to damage identification of both simulated simple beams and a test simple beam. The results show that the indicator with elimination of noise effects is able to locate the damaged parts in the investigated beams.

Value choosing of parameters in stochastic subspace identification

There have been many papers related to stochastic subspace identification (SSI). The principle of SSI can be referred to reference (Peeters, 2000).

Determination of the order of system is usually regarded as most important in the modal parameter identification using SSI. However, it was found by simulations of simple beams that there were relations among the row block number *i* of Hankel matrix, the order of system *n* and the signal noise ratio (SNR). So the value of *i* is also important to the modal parameter identification. But there has been no reported work to prove it so far. Analyses on choosing values of *i* and *n* will be carried out as follows.

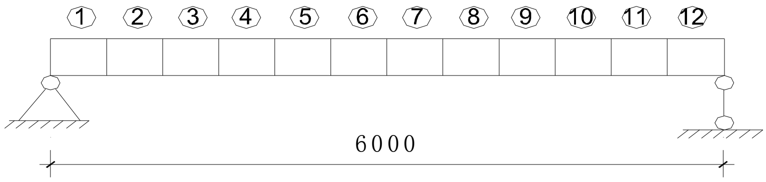


Figure 1. The finite element model of a simple beam

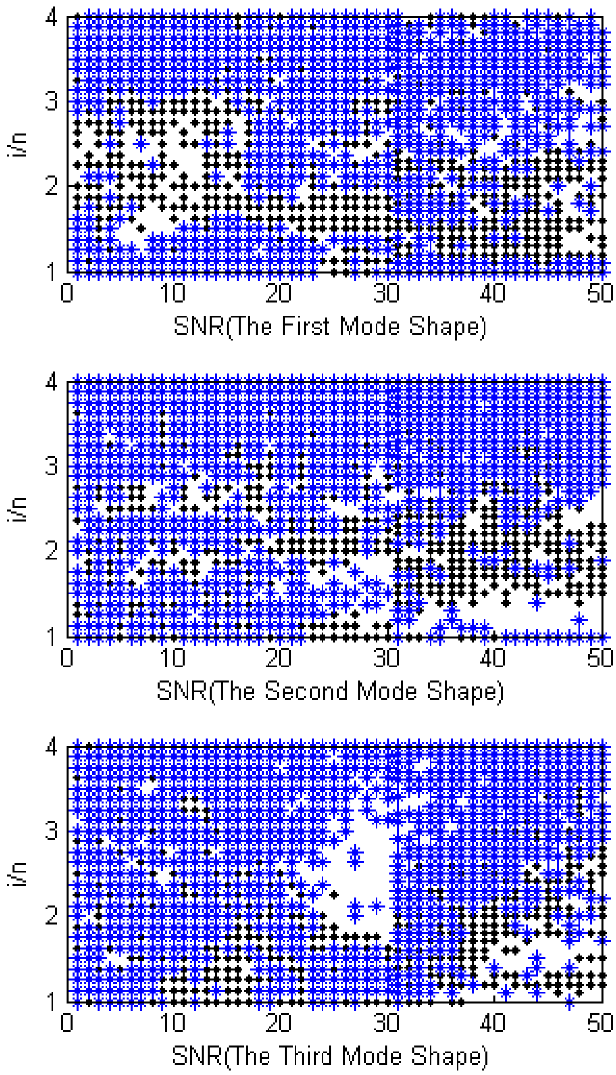


Figure 2. The relation between *i*/*n* and SNR

A simple beam model was built in ANSYS, of which the configuration is shown in Figure 1. The cross section dimension was 0.25×0.20 m^2, the span length was 6m. The Young's modulus was 32Gpa and the density was 2500 kg/ m^3. The beam was divided into 12 elements uniformly and the nodes were numbered from the left to the right.

The vibration modes of the beam were obtained by modal analysis in ANSYS and by SSI respectively. The former is called exact modes. As for the latter, the following steps are adopted.

1) Accelerations at each node were calculated by dynamic analysis in ANSYS.

2) The accelerations were added with white noise of different amplitude (Cao and Lin, 2010).

3) The noise-polluted accelerations were used as signals to identify structural modes with SSI. In the process, *i* was taken as different values with signals of different SNR to get different modes.

4) Comparing the modes identified with different values of *i* with the corresponding exact modes, the values of *i* of the modes the most related and the least related to the exact mode were called TM and TL respectively.

5) Using SNR as the abscissa, the value of *i*/*n* as the ordinate, the values of *i*/*n* corresponding to TM and TL were plot with black dots and blue asterisks respectively.

The above steps were repeated 10 times considering the random property of noise.

As shown in Figure 2, it is easy to find the relations of *i*/*n* and SNR by the distribution of the blue asterisks and the black dots. For the first three modes, with SNR being around 40dB, when *i*/*n* is taken as 1.2~2.2, 1.4~2.2 and 1.2~2.0 respectively, the modes related to the corresponding exact modes well. So it is suggested that, when SNR is about 40dB, *i*/*n* is taken as 1.5~2.0 to get the best results.

Reconstruction of structural modal rotations

The dynamic equilibrium equation of a bending-type structure is expressed as

$$\begin{bmatrix} M & \\ & J \end{bmatrix}\begin{Bmatrix} \ddot{x} \\ \ddot{\varphi} \end{Bmatrix}+\begin{bmatrix} C_{xx} & C_{x\varphi} \\ C_{\varphi x} & C_{\varphi\varphi} \end{bmatrix}\begin{Bmatrix} \dot{x} \\ \dot{\varphi} \end{Bmatrix}+\begin{bmatrix} K_{xx} & K_{x\varphi} \\ K_{\varphi x} & K_{\varphi\varphi} \end{bmatrix}\begin{Bmatrix} x \\ \varphi \end{Bmatrix}=\begin{Bmatrix} f(t) \\ 0 \end{Bmatrix} \tag{1}$$

The relation of rotational vector and translational vector can be easily obtained by eq.1

$$\varphi=-K_{\varphi\varphi}^{-1}K_{\varphi x}x \tag{2}$$

The global stiffness matrix is expressed as

$$K_i=\sum_{i=1}^{n}b_i\overline{K}_i \tag{3}$$

Where, b_i is the parameter of the i[th] element to be identified. $\overline{K}_i$ is the elemental stiffness matrix with the parameter b_i being extracted.

Similarly, for the respective block matrix, the following holds,

$$K_{\varphi x}=\sum_{i=1}^{n}b_i\overline{K}_{\varphi x}^{(i)} \tag{4}$$

$$K_{\varphi\varphi}=\sum_{i=1}^{n}b_i\overline{K}_{\varphi\varphi}^{(i)} \tag{5}$$

in which,

$$B=(b_1I_n,b_2I_n,\ldots,b_nI_n) \tag{6}$$

n is the number of elements, I_n is $n\times n$ unit matrix.

$$\overline{K}_{\varphi x}=(\overline{K}_{\varphi x}^{(1)},\overline{K}_{\varphi x}^{(2)},\ldots,\overline{K}_{\varphi x}^{(n)})^T \tag{7}$$

$$\overline{K}_{\varphi\varphi}=(\overline{K}_{\varphi\varphi}^{(1)},\overline{K}_{\varphi\varphi}^{(2)},\ldots,\overline{K}_{\varphi\varphi}^{(n)})^T \tag{8}$$

So we have

$$K_{\varphi x}=B\overline{K}_{\varphi x} \tag{9}$$

$$K_{\varphi\varphi}=B\overline{K}_{\varphi\varphi} \tag{10}$$

Eq.2 can be rewritten as,

$$\varphi = -(B\overline{K}_{\varphi\varphi})^{-1}\overline{K}_{\varphi x}x \tag{11}$$

By the above steps, the modal rotations can be evaluated by modal translations.

Damage indicator based on element modal strain energy and data fusion theory

Element damage variable

Liu et al (2004) constructed a damage indicator based on element modal strain energy (EMSE), called element damage variable. The element damage variable of the j^{th} element is expressed as

$$D_j = \frac{\left|EMSE_j^d - EMSE_j^u\right|}{\left|EMSE_j^d - EMSE_j^u\right| + EMSE_j^u} \tag{12}$$

Where $EMSE_{ij}^u$ and $EMSE_{ij}^d$ are EMSEs of the j^{th} element in intact and damaged state respectively. And are expressed as,

$$EMSE_j^u = \sum_{i=1}^{m} \varphi_i^T K_j \varphi_i \tag{13}$$

$$EMSE_j^d = \sum_{i=1}^{m} \tilde{\varphi}_i^T K_j \tilde{\varphi}_i \tag{14}$$

In which, K_j is the stiffness matrix of the j^{th} element, φ_i and $\tilde{\varphi}_i$ are the i^{th} mode under intact and damaged state respectively.

Since damage leads to reduction of structural stiffness, the value of EMSE of damaged state should be larger than that of intact state calculated by Eq.13 and Eq.14 respectively. If the numerator in Eq.12 is not taken the absolute value, as in Eq.15, the damaged element will always give positive D while those intact elements will often do reversely. So by both the value and sign of D, it is much easier to identify the damaged elements, which has been verified by the authors (Cao et al, 2008).

$$D_j = \frac{EMSE_j^d - EMSE_j^u}{\left|EMSE_j^d - EMSE_j^u\right| + EMSE_j^u} \tag{15}$$

Multi-source information fusion

Multi-source information fusion or multi-sensor information fusion is a new technology which has been developed since 70s in the last century (Waltz 1990; Linn et al, 1991; Hall, 1992; Kang, 1997; Yang, 2004; Wan et al, 2005; Han et al, 2006; Liu et al, 2008). The principle is that information obtained by data fusion from several sensors is more useful than that from only one sensor. The main techniques involved include classic derivation and statistics, Byes derivation, Dempster-Shafer evidence theory, fuzzy theory, etc. In this paper , Dempster-Shafer evidence theory (D-S theory in short) (Shafer, 1976; Han et al, 2006) is used to improve the indicator based on EMSE.

The basic idea of D-S theory is as follows.

Assume that $A_1, A_2, \cdots, A_n$ are n incompatible events, $D_1, D_2, \cdots, D_m$ are m sensors. The occurrence probability of the j^{th} event by the i^{th} sensor is $M_i(A_j)$, then the occurrence probability of event P is,

$$M(P) = C^{-1} \sum_{\cap A_j = p} \prod_{1 \le i \le m} M_i(A_j) \tag{16}$$

in which,

$$C = 1 - \sum_{\cap A_j = \Phi} \prod_{1 \le i \le m} M_i(A_j) = \sum_{\cap A_j \ne \Phi} \prod_{1 \le i \le m} M_i(A_j) \tag{17}$$

Damage indicator

Previously, the value of number *i* of Hankel matrix row block was suggested to ensure exact extraction of modes by SSI. However, the extracted modes are still influenced by noise. So the D-S evidence theory will be used to carry out data fusion for results obtained by different values of *i* to eliminate noise as nearly as possible.

Firstly, the change of EMSE is defined as,

$$LEMSE_{ij} = \frac{D_j}{\sum_j D_j} \tag{18}$$

in which, D_j is calculated by Eq. 15.

That *n* elements being damaged is thought as *n* events. The values of LEMSE corresponding to *m* values of *i* are regarded as information given by *m* sensors. By eq.16, the damage indicator based on EMSE and data fusion theory is,

$$LEMSE_p = C^{-1} \sum_{\cap j=p} \prod_{1\le i\le m} LEMSE_{ij} \tag{19}$$

in which

$$C = 1 - \left| \sum_{\cap j=\Phi} \prod_{1\le i\le m} LEMSE_{ij} \right| = \left| \sum_{\cap j\ne\Phi} \prod_{1\le i\le m} LEMSE_{ij} \right| \tag{20}$$

P means the p^{th} element.

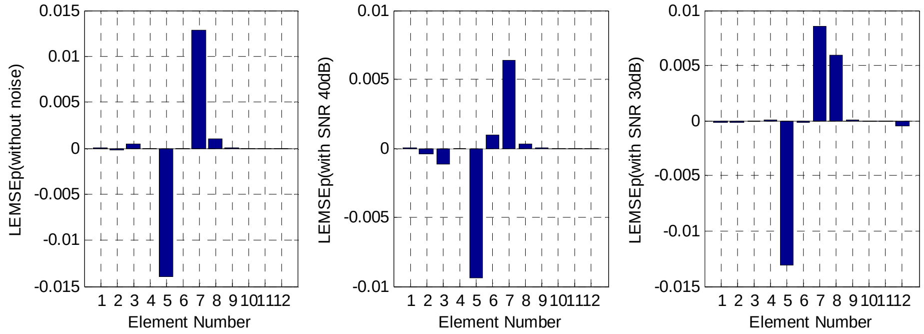
Simple beam simulations

A simple beam simulation was used to verify the suggested damage indicator. The simple beam was the same as that in Figure 1. Damage of element was simulated by the reduction of element stiffness. The cases of damage see table 1. With white noise excitations being applied at the 4^{th} node, acceleration responses at each node of intact and damaged state were calculated respectively. Noise was added into the responses with two SNR correspondingly, i.e. 40dB and 30dB. SSI was used to extract the first three modes of the beam under intact and damaged state respectively from the noised responses. Then the modal rotations were estimated by eq.11. Both the modal translations and rotations were utilized to construct the damage indicator by eq.19. The results are shown in (a) to (e) of Figure 3.

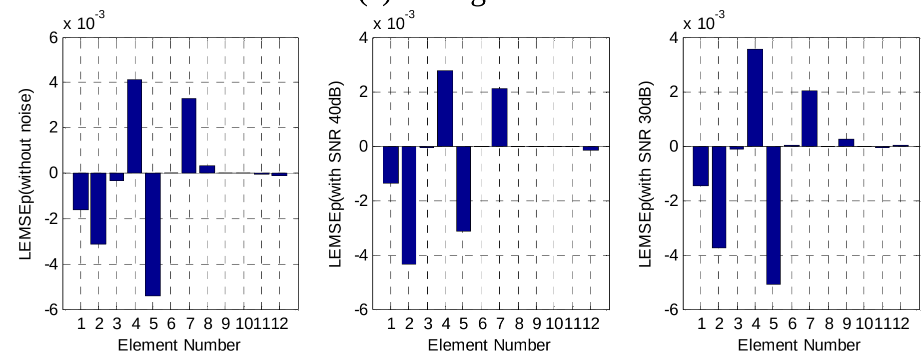
In single damage cases, the suggested damage indicator can locate the damaged elements even under 30dB noise. However, the adjacent elements are wrongly identified as damaged. In two damage cases, the element with less damage can be identified, but the value of damage indicator is close to those of wrongly identified elements. If the two elements have damage of the same degree, the results are better. So it is possible to identify damage of a simple beam with the suggested damage indicator. Even under heavy noise with SNR being 30dB, the damage elements can still be located.

Table 1. Damage cases of the simple beam

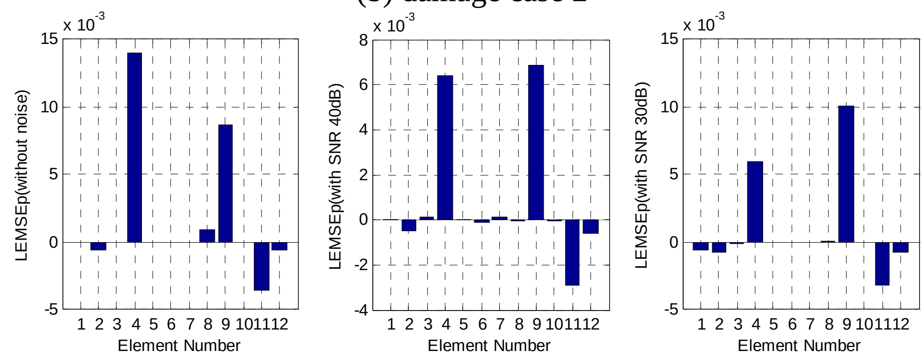
Damage case	Damaged elements	Damaged extent
1	7	7-10%
2	4、7（non-symmetric）	4-10%、7-10%
3	4、9 (symmetric）	4-10%、9-10%
4	4、7（non-symmetric）	4-10%、7-20%
5	4、7、9	4-10%、7-10%、9-10%



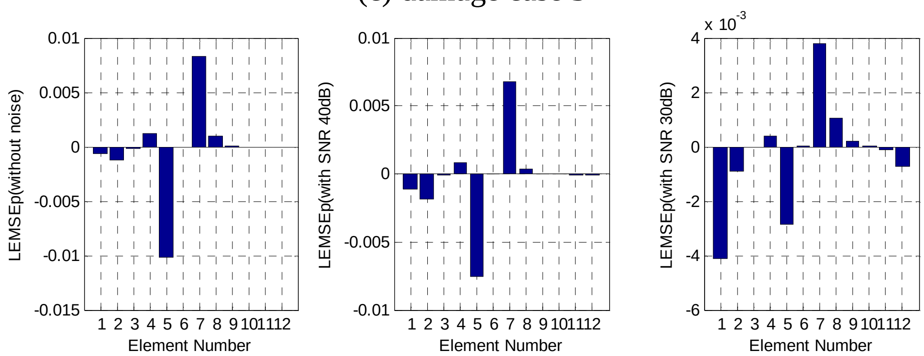
(a) damage case 1



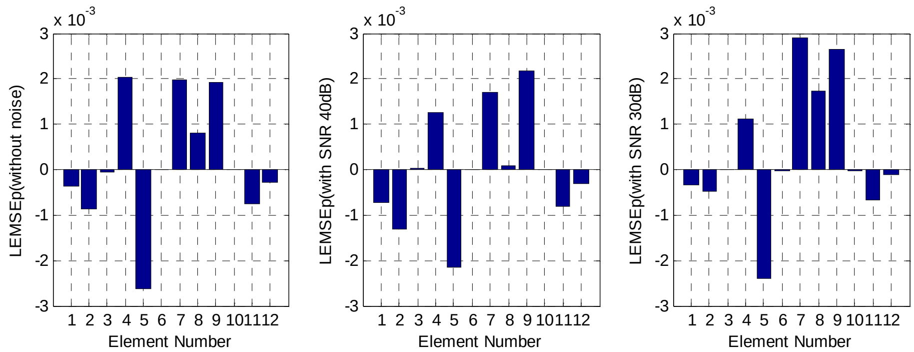
(b) damage case 2



(c) damage case 3



(d) damage case 4



(e) damage case 5

Figure 3. Damage detection of the simple beam under different noise levels

The test simple beam

The test beam was a simple reinforced concrete beam, with section dimension being 210x190mm, and span length being 4.5m. Three steel bars and two steel bars, with diameter being 12mm, were uniformly distributed in the tension side and compression side respectively. The confined steel was Φ8@225. The thickness of concrete cover was 20mm. Nine acceleration sensors were evenly distributed on the top of the beam, dividing the beam into ten segments, see Figure 4.

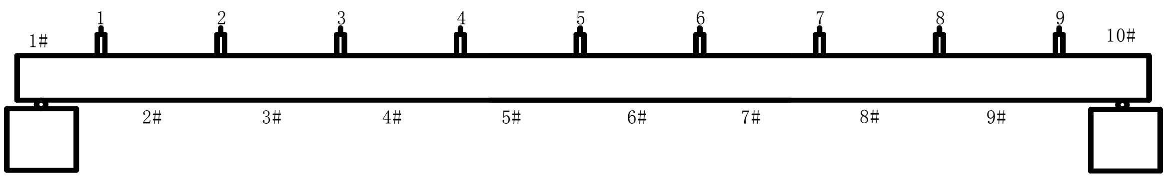
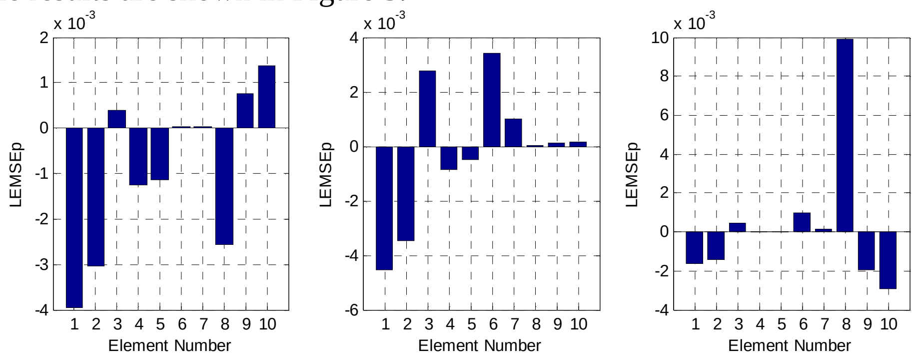


Figure 4. The layout of acceleration sensors on the test beam

Firstly, the beam was excited by a wood hammer. The accelerations of free vibration were recorded by the nine sensors. Then damage in the beam were made by cutting a slot. The slot was of U shape, on both sides and the bottom of the beam, 20mm wide and 20mm deep. After the first slot was cut in the middle of the 3rd segment, the beam was excited and accelerations were recorded. And then the second slot was cut in the middle of the 6th segment and accelerations of free vibration of the beam were also recorded. At last, in the middle of the 8th segment the last slot was cut and accelerations were recorded.

With the recorded accelerations, damage indicators were constructed by the procedure suggested previously. The results are shown in Figure 5.



(a) damage in the 3ird element (the first cut) (b) damage in the 3ird and 6th elements (the second cut) (c) damage in the 3ird, 6th and 8th elements (the third cut)

Figure 5. Damage detection of the test beam

From Figure 5, after the first cutting was made, the suggested damage indicator could locate the damaged segment, i.e. the 3rd segment. However, the 9th and 10th segment were wrongly identified as damaged. By checking the original signals recorded by the 8th and 9th sensors, the signals were found with heavier noise than the others. It is estimated that the two sensor were not adhesive tightly to the top of the beam when the first dynamic test was made. And maybe they were not as tight as should be in the second dynamic test. When the 6th segment and 8th segment were damaged, the damage indicator could also locate them, although the 7th segment was wrongly recognized as damaged.

Conclusions

Structural modal translations were extracted from accelerations by stochastic subspace identification. Modal rotations were reconstructed by modal translations using the principle of static condensation. Both modal translations and rotations were used to calculate the change of element modal strain energy, which was then utilized to construct a damage indicator based on the theory of data fusion. Analyses were also carried out on choosing values of parameters in SSI for eliminating the effect of noise. The constructed damage indicator was applied to damage identification of both simulated simple beams and a test simple beam.

The results of simulations show that the suggested damage indicator could locate the damaged elements of simple beams even under heavy noise in one damaged element cases, although the adjacent elements were wrongly recognized as damaged. In multi-damage cases, the less damaged element could not be detected when the difference of damaged level between any two elements is higher than the damaged level of the less damaged element.

As for the test simple beam, the damage indicator could identify the damaged segments after the slots were cut sequentially. The adjacent segment was wrongly identified, but its value of damage indicator was smaller than that of damaged segments.

It can be seen from the results from simulation and test that the suggested damage indicator can locate the position of damage of beam-type structures. It is sensitive to small damage. Moreover, it is able to resist noise at a considerable level.

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