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On Wavelet-based Arithmetic of Closure Solutions to Nonlinear Differential Equations

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Since the nonlinear science was initiated in the last century, the nonlinear investigations have extensively entered in most scientific areas and even in some engineering applications. Unlike the linear cases, there are no well-rounded theoretical and numerical approaches in solving a general nonlinear problem. The efficiency of the approaches usually depends on nonlinear features of the problem to be solved, e.g., most solution methods referring to perturbations, iterations, and series expansions etc. are mainly appropriate to problems with weak nonlinearity. For solving strong nonlinear problems, some special techniques must be employed to each case one-by-one. Meanwhile, almost all solutions obtained by these quantitative techniques are not closed. In other words, the solutions are dependent on those truncated desertions. In this study, we introduce a new technique for generally solving the initial- and boundary-value problems with arbitrary nonlinear features no matter weak or strong. When applying such a technique, the Galerkin method is employed after using the scaling function transform in wavelet theory to approximate unknown functions in the differential equations with initial or/and boundary conditions. More specifically, first of all, we propose a new boundary-expansion technique for functions on bounded intervals so that the significant jumping error at each boundary, appeared during their reconstructions, can be eliminated. Next, we propose a new calculation technique to treat the decomposition or expansion coefficients in high accuracy. Eventually, terms with any types of nonlinearity associated with the unknown functions in the equations can be explicitly expressed in closure forms of the coefficients which can be determined by a set of nonlinear algebraic equations as the Galerkin method is applied, where the independence of the expansion coefficients within the solution interval on those truncated is evidently exhibited. Finally, solutions of some quantitative examples on mechanics problems with strong nonlinearity are demonstrated by using this new arithmetic.

Keywords: Nonlinear differential equations, Wavelet-based solution arithmetic, Closure feature, Weak or strong nonlinearity, Application examples.