

## **A Coupling Method using FEM of 2D Shallow Water Flow and 3D Gas-Liquid Two-Phase Flow**

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A coupling method based on an implicit mixed finite element method of two-dimensional shallow water long wave equations and three-dimensional Navier-Stokes equations is proposed in this paper. A stabilized MINI element, bubble function element/linear element, with an implicit scheme in time is used to solve the shallow water long wave equations and the Navier-Stokes equations. The stabilized MINI element means bubble function element stabilization method for triangular element and tetrahedral element. The estimation of gas-liquid interface for three-dimensional Navier-Stokes equations is employed an interface-capturing method based on a phase-field model (PFM). Cahn-Hilliard equation in PFM is applied to estimate the interface of gas and liquid. The stabilized MINI element is used to solve Cahn-Hilliard equation. Compatibility condition and equilibrium condition are considered for connecting the two-dimensional shallow water long wave equations and three-dimensional Navier-Stokes equations. A solitary wave problem is analyzed to verify presented coupling method as a computational example.

**Keywords:** Coupling method, 2D Shallow water flow, 3D Gas-liquid two-phase flow, Finite element method, MINI element