

## **Calculation of hydrodynamic forces for unsteady Stokes flows by singularity integral equations based on fundamental solutions**

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The attractive feature of the singularity method or the method of fundamental solutions for steady Stokes flows is that the hydrodynamic forces acting on the particle can be calculated by the total strength of distributed singularities. For unsteady Stokes flows, however we have to derive hydrodynamic forces acting on a solid body in terms of the strengths of both unsteady Stokeslets as well as unsteady potential dipoles if mass and force sources are both taken into consideration. Since the hydrodynamic force formulation results in a Volterra integral equation of the first kind, the strengths are numerically approximated by means of the Lubich convolution quadrature method (CQM) in this study. As far as the numerical solutions of time-domain integral formulations of the unsteady Stokes equations are concerned, this paper requires only the Laplace-domain instead of the time-domain fundamental solutions of the governing equations. The stability and accuracy of the proposed method are verified through some well selected numerical examples. In total they include two examples presenting the accuracy of Lubich CQM, and another two examples for calculating general hydrodynamic forces of a sphere in oscillating or non-oscillating unsteady Stokes flows. It is concluded that this study is able to extend the unsteady Stokes flow theory to more general transient motions instead to limit to the oscillating flows only.

**Keywords:** Fundamental solutions, Stokeslets, dipole, Volterra integral equation, convolution quadrature method