Entropically Damped Artificial Compressibility for SPH

Prabhu Ramachandran [*]
Department of Aerospace Engineering, IIT Bombay
Powai, Mumbai 400076
prabhu@aero.iitb.ac.in

Kunal Puri
Department of Aerospace Engineering, IIT Bombay
Powai, Mumbai 400076
kunal.r.puri@gmail.com

March 31, 2015

Abstract

The Entropically Damped Artificial Compressibility (EDAC) method of Clausen [2013] is an interesting alternative to the traditional artifical compressibility technique. We apply the EDAC method in the context of Smoothed Particle Hydrodynamics. In this work we present preliminary results for this method. The method is easy to implement in a standard SPH framework. Our simulations are compared with those of the Transport Velocity Formulation of Adami and Hu [2013] as well as a standard SPH formulation where necessary.

The results appear very promising and seem comparable to those of the TVF method. Like the TVF, it does not produce spurious pressure oscillations. Unlike the TVF, which cannot be directly applied to free-surface problems, the new method has no such difficulty.

Keywords: SPH, Incompressible flow, Artificial compressibility, Entropically Damped Artificial Compressibility

1 Introduction

The Smoothed Particle Hydrodynamics method has been applied to a wide variety of problems including elastic dynamics, compressible fluid flow, viscous incompressible fluid flow, multiphase problems etc. For incompressible flows, SPH implementations either resort to a weakly-compressible formulation using the artificial compressibility technique or use a pressure-based approach as seen in several incompressible SPH schemes. The difficulty with the incompressible projection based schemes is the need to solve an implicit problem which poses computational challenges when scaling up problems to larger sizes and involves increased complexity. The weakly-compressible formulation, while explicit, faces problems related to unphysical pressure oscillations. The pressure oscillations are reduced greatly when the Transport Velocity Formulation of Adami et al. [2013] (TVF) is used. The method introduces a background pressure that serves to reduce tensile instability and reduces pressure oscillations. In the words of the

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[*] Address all correspondence to this author.

authors, the method produces “unprecedented accuracy and stability”. The difficulty with the TVF formulation is that it cannot handle free-surface problems directly on account of the background pressure. The ability to naturally capture free-surface problems is a desired feature of the SPH method.

Recently, the Entropically Damped Entropically Damped Artificially Compressible method of Clausen [2013a,b] (EDAC) has been applied to finite-difference and finite-element schemes. The method employs a new approach and evolves the pressure in time. This evolution equation eliminates the need to use the stiff equation of state that is usually employed.

In the present work, we combine the Entropically Damped Artificially Compressible method with the SPH formulation and the boundary condition of Adami et al. [2012]. This results in a simple set of equations and produces results that are roughly comparable to that of the TVF. The method can also handle free-surfaces as it does not require a background pressure. The results for the dam break problem appear to be good.

We perform simulations for a few classic problems and show that the method produces stable and accurate results comparable to the best available SPH schemes albeit with much simpler equations and no additional corrections. To our knowledge this is the first time that this technique has been applied to the SPH.

2 The Numerical Method

As discussed in Clausen [2013a], the basic idea behind the EDAC method is to introduce an evolution equation for the pressure, p, instead of an equation of state. The evolution equations for momentum and pressure are written as,

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \mathrm{grad}(\mathbf{u}) = -\frac{1}{\rho}\nabla p + \mathrm{div}(\sigma), \tag{1}$$

$$\frac{\partial p}{\partial t} + \mathbf{u} \cdot \mathrm{grad}(p) = -\rho c_s^2 \mathrm{div}(\mathbf{u}) + \nu \nabla^2 p \tag{2}$$

In the equations above, $\mathbf{u}$ is the velocity of the fluid, p is the pressure σ represents the deviatoric part of the stress tensor, c_s is the speed of sound (which is set to be a multiple of the maximum speed of the fluid). ν is the kinematic viscosity of the fluid. The density, ρ is held constant. We start with a zero pressure and the pressure evolves naturally from the equation (2) above.

3 Numerical Implementation

In order to simulate the equations above with the SPH, the following SPH discretizations are used. The density, ρ is held constant. The momentum equation is discretized using the standard SPH approach using Monaghan's original formulation but without any artificial viscosity,

$$\frac{d\mathbf{u_i}}{dt} = -\sum_j m_j \left(\frac{p_j}{\rho_j^2} + \frac{p_i}{\rho_i^2}\right) \nabla W_{ij} \tag{3}$$

We add a real viscosity based on the Morris formulation. For equation (2) we use the following,

$$\frac{dp_i}{dt} = \sum_j m_j c_s^2 \, \mathbf{u_{ij}} \cdot \nabla W_{ij} + 2\frac{m_j}{\rho_j} \nu (p_i - p_j) \frac{\mathbf{u_{ij}} \cdot \mathbf{r_{ij}}}{r_{ij}^2 + \eta} \tag{4}$$

For the boundary conditions, we employ the formulation of Adami et al. [2012].

4 Results

Using the equations above, we simulate a few standard problems and present the results as compared with other SPH formulations.

4.1 Dam break

The first test case we consider is the traditional two-dimensional dam-break problem.

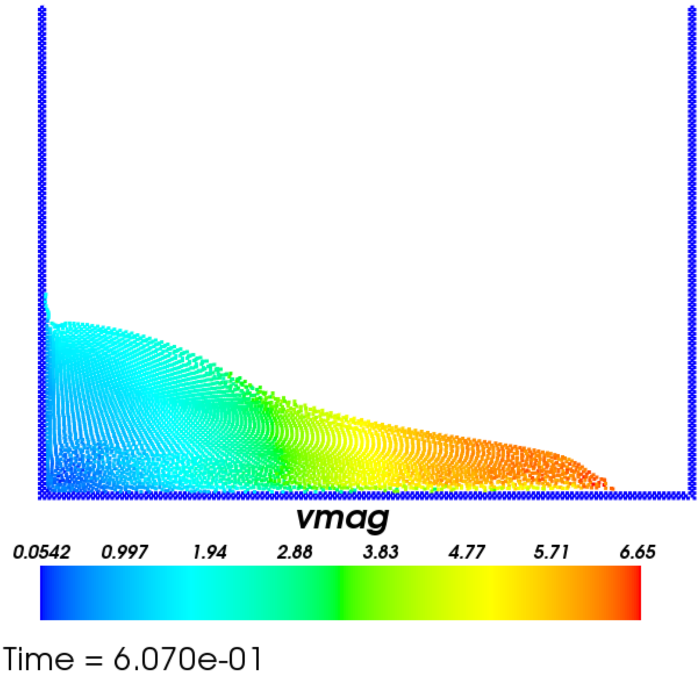

Figure 1: Two-dimensional dam-break problem solved with the EDAC-SPH method.

As can be seen, there is much less fluid sticking to the walls on the side and the results are comparable to those produced by the traditional SPH technique.

We have obtained good results for a variety of other problems like Couette flow, Poiseuille flow, Lid-Driven-Cavity and the Taylor Green vortex. We are in the process of comparing these results with other SPH schemes.

5 Conclusions

We have shown preliminary results of a new scheme that combines the EDAC method with SPH. The resulting formulation is very simple, does not display spurious pressure oscillations, does not require a background pressure and seems to produce good results for a few test problems. We will be performing more rigorous tests and will compare the new method carefully with results using the TVF and the standard SPH.

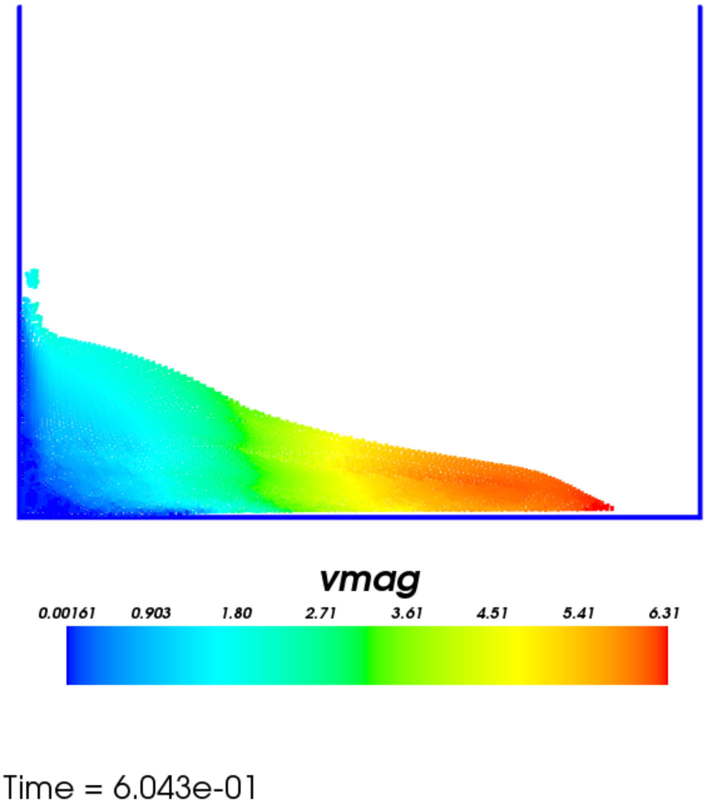

Figure 2: Two-dimensional dam-break problem solved with the traditional SPH.

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