Three-dimensional welding residual stresses evaluation based on the eigen-strain methodology via X-ray measurements at the surface

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Abstract

In order to assure structural integrity for operating welded structures, it is necessary to evaluate crack growth rate and crack propagation direction for each observed crack nondestructively. Here, three dimensional welding residual stresses must be evaluated to predict crack propagation. Today, X-ray diffraction is used and the ultrasonic method has been proposed as nondestructive method to measure residual stresses. However, it is impossible to determine residual stress distributions in the thickness direction. Although residual stresses through a depth of several tens of millimeters can be evaluated nondestructively by neutron diffraction, it cannot be used as an on-site measurement technique. It is because neutron diffraction is available only in special irradiation facilities. Author pays attention to the bead flush method based on the eigen-strain methodology. In this method, three-dimensional welding residual stresses are calculated by an elastic FEM (Finite Element Method) analysis from eigen-strains which are evaluated by an inverse analysis from released strains by strain gauges in removal of reinforcement of weld. Here, the removal of the excess metal can be regarded as nondestructive treatment essentially because toe of weld which may become crack starters can be eliminated. The effectiveness of the method has been proved for welded plates and pipes even with relatively lower bead height. In actual measurements, stress evaluation accuracy becomes poorer because measured values of strain gauges are affected by processing strains on the machined surface. In the previous studies, the author has developed the bead flush method that is free from the influence of the affecting strains by using residual strains on surface by X-ray diffraction. However, stress evaluation accuracy is not good enough because of relatively poor measurement accuracy of X-ray diffraction. In this study, a method to improve the estimation accuracy of residual stresses in this method is formulated, and it is shown numerically that inner welding residual stresses can be estimated accurately from the measured residual strains by X-ray diffraction.

Keywords: Eigenstrain, Weld, Residual stress, X-ray diffraction, Bead flush method, Three-dimensional evaluation,

Introduction

In order to assess structural integrity for operating welded structures, it is important to evaluate three-dimensional welding residual stresses non-destructively to predict crack propagating for observed cracks in in-service inspection. Today, there are some techniques to estimate three-dimensional residual stresses such as neutron diffraction methods [Suzuki and Akita (2009)], welding simulation via thermal elastic-plastic FEM analysis [Yaghi et al. (2013)] and techniques based on the eigen-strain methodology [Mura (1978)]. However, neutron diffraction is unavailable to use as an on-site measurement application because it can be used only in special irradiation facilities. Furthermore, measured stresses from diffraction methods including X-ray diffraction and high energy X-ray diffraction techniques cannot be input into the FEM model that has been used in assessment of structural integrity at the time of the design. It is because all the 6 stress components that satisfy the self-equilibrium condition cannot be measured. Also, it is difficult to predict crack propagation via FEM [Kikuchi et al. (2009)] from estimated residual stresses by diffraction methods. Although welding residual stresses can be estimated non-destructively by using welding simulation, estimation accuracy may be poorer due to the difficulty of determining the parameters depended on temperature. To make matters worse, piece-to-piece variations have to be neglected in qualitative evaluation via thermal elastic-plastic FEM analysis. On the other hand, three-dimensional residual stress distribution can be estimated quantitatively by FEM analysis by using the eigen-strain methodology. For example, the cutting method [Ueda et al. (1975); Ueda et al. (1979)] based on the eigen-strain methodology has been proposed. In this method, residual stresses are determined by an elastic FEM analysis from eigen-strains which are calculated by an inverse analysis [Kubo (1992)] from released strains through sectioning. Here, eigen-strains are defined as a sum of inelastic strains [Mura (1978)] and can be regarded as the cause of residual stresses and elastic strains. Note that they are not always equal to inherent strains which are a total of physical inelastic strains such as thermal, plastic and transformation strains [Masuda and Nakamura (2010a; 2010b)]. Although structures have to be wasted by the cutting method, welding residual stresses can be evaluated non-destructively by the bead flush method [Nakamura et al. (1995)]. In this method, eigen-strains are estimated from released strains in removal of reinforcement of weld. Since toe of weld may become crack starters, the removal of the excess weld metal can be regarded as a preferable treatment. The effectiveness of this method has been proved numerically for welded plates [Kumagai et al. (2000)]. In addition, statistical range of residual stress distributions has been accumulated successfully for welded pipes even with lower bead height [Ogawa and Nakamura (2011a; 2011b)]. In actual measurement, however, processing strains are created after machining the reinforcement of the weld. In this case, stress evaluation accuracy becomes poorer because measured values of strain gauges are affected by the processing strains. In order to solve the difficulties, the bead flush method has been developed to be free from the influence of the affecting strains [Ogawa (2013)]. In this method, not only welding eigen-strains but also processing strains are estimated non-destructively from residual strains on surface by X-ray diffraction instead of released strains by strain gauges (Fig. 1). However, estimation accuracy in this method is not higher due to relatively poor measurement accuracy of X-ray diffraction.

In this study, numerical formula to be able to use the measured residual strains on the weld metal after the removal as additional source of information is shown. And, numerical simulation is carried out to prove the effectiveness in this method.

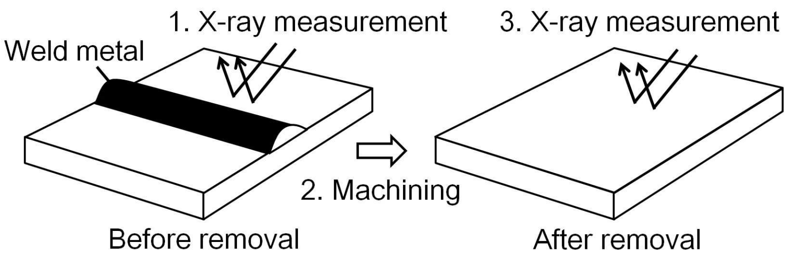


Figure 1. Procedures in the advanced bead flush method

Analytical Procedures

Formulation of the Bead Flush Method

In general, the elastic strains $\{\boldsymbol{\varepsilon}_\mathrm{e}\}$ of the concerned elements and the eigen-strains $\{\boldsymbol{\varepsilon}_\mathrm{e}^*\}$ can be related as:

$$\{\boldsymbol{\varepsilon}_\mathrm{e}\} = [\boldsymbol{R}_\mathrm{e}]\{\boldsymbol{\varepsilon}_\mathrm{e}^*\} \tag{1}$$

where $[\boldsymbol{R}_\mathrm{e}]$ is an elastic response matrix. And, the *i*-th column of it can be obtained by imposing an unit eigen-strain vector to an *i*-th component of $\{\boldsymbol{\varepsilon}_\mathrm{e}^*\}$ as shown below:

$$\{_\mathrm{unit}\,\boldsymbol{\varepsilon}_\mathrm{e}^*\}_i = \{0,\cdots,\boldsymbol{\varepsilon}_{\mathrm{e}i}^* = 1,\cdots,0\}^\mathrm{T} \tag{2}$$

Therefore, elastic strains before and after removals of excess metal can be described as follows:

$$\{\boldsymbol{\varepsilon}_\mathrm{eb}\} = [\boldsymbol{R}_\mathrm{eb}]\{\boldsymbol{\varepsilon}_\mathrm{eb}^*\} \tag{3}$$

$$\{\boldsymbol{\varepsilon}_\mathrm{ea}\} = [\boldsymbol{R}_\mathrm{ea}]\{\boldsymbol{\varepsilon}_\mathrm{ea}^*\} \tag{4}$$

where the subscripts b and a denote the before and after removals, respectively. Since it is based on the assumption that eigen-strains are constant through machining, the released strain vectors $\{\boldsymbol{\Delta\varepsilon}_\mathrm{e}\}$ are given by the following equations:

$$\begin{aligned}\{\boldsymbol{\Delta\varepsilon}_\mathrm{e}\} &= \{\boldsymbol{\varepsilon}_\mathrm{ea}\} - \{\boldsymbol{\varepsilon}_\mathrm{eb}\}\\ &= ([\boldsymbol{R}_\mathrm{a}] - [\boldsymbol{R}_\mathrm{b}])\{\boldsymbol{\varepsilon}^*\}\\ &= [\boldsymbol{R}]\{\boldsymbol{\varepsilon}^*\}\end{aligned} \tag{5}$$

where $[\boldsymbol{R}]=[\boldsymbol{R}_\mathrm{a}]-[\boldsymbol{R}_\mathrm{b}]$ and $\{\boldsymbol{\varepsilon}^*\}=\{\boldsymbol{\varepsilon}_\mathrm{b}^*\}=\{\boldsymbol{\varepsilon}_\mathrm{a}^*\}$. In actual measurements, measured released stains by strain gauges include measurement errors $\{\boldsymbol{\Delta\varepsilon}_\mathrm{err}\}$. In this case, measured released strain vector $\{\boldsymbol{\Delta\varepsilon}_\mathrm{em}\}$ is written as follows:

$$\{\boldsymbol{\Delta\varepsilon}_\mathrm{em}\} = [\boldsymbol{R}]\{\boldsymbol{\varepsilon}^*\} + \{\boldsymbol{\Delta\varepsilon}_\mathrm{err}\} \tag{6}$$

The most probable values of estimated eigen-strain vector $\{\boldsymbol{\varepsilon}_\mathrm{est}^*\}$ is described by the least square method as follows:

$$\{\boldsymbol{\varepsilon}_\mathrm{est}^*\} = [\boldsymbol{R}]^+\{\boldsymbol{\Delta\varepsilon}_\mathrm{em}\} \tag{7}$$

where $[\boldsymbol{R}]^+$ is the Moore and Penrose generalized inverse matrix [Kubo (1992)] of $[\boldsymbol{R}]$, and it is written as:

$$[\boldsymbol{R}]^+ = [\boldsymbol{R}]^\mathrm{T}[\boldsymbol{R}]([\boldsymbol{R}]^\mathrm{T}[\boldsymbol{R}][\boldsymbol{R}]^\mathrm{T}[\boldsymbol{R}])^-[\boldsymbol{R}]^\mathrm{T} \tag{8}$$

Improvement of the Bead Flush Method

In the conventional bead flush method, excess metal has to be eliminated without affecting strains. Once processing strains $\{\boldsymbol{\varepsilon}_\mathrm{p}^*\}$ are created on a sample, measurement accuracy of release stains is worsened as shown below:

$$\{\boldsymbol{\Delta\varepsilon}_\mathrm{em}\} = [\boldsymbol{R}]\{\boldsymbol{\varepsilon}^*\} + \{\boldsymbol{\Delta\varepsilon}_\mathrm{err}\} + [\boldsymbol{R}_\mathrm{a}]\{\boldsymbol{\varepsilon}_\mathrm{p}^*\} \tag{9}$$

In order to improve this problem, the author has proposed the following equations instead of Eq. (7) [Ogawa (2013)].

$$\{\boldsymbol{\varepsilon}_\mathrm{est}^*\boldsymbol{\varepsilon}_\mathrm{p\_est}^*\}^\mathrm{T} = [\boldsymbol{R}_\mathrm{ab}]^+\{\boldsymbol{\varepsilon}_\mathrm{ebm}\boldsymbol{\varepsilon}_\mathrm{eam}\}^\mathrm{T} \tag{10}$$

$$[\boldsymbol{R}_\mathrm{ab}] = \begin{bmatrix}\boldsymbol{R}_\mathrm{b} & \boldsymbol{0}\\ \boldsymbol{R}_\mathrm{a} & \boldsymbol{R}_\mathrm{a}\end{bmatrix} \tag{11}$$

where $\{\boldsymbol{\varepsilon}_\mathrm{ebm}\}$ and $\{\boldsymbol{\varepsilon}_\mathrm{eam}\}$ are measured residual strains before and after removals, respectively. And, these two residual strains can be measured non-destructively by X-ray diffraction. Therefore, it is possible to obtain estimated values of both welding eigen-strains $\{\boldsymbol{\varepsilon}_{\mathrm{est}}^{*}\}$ and processing strains $\{\boldsymbol{\varepsilon}_{\mathrm{p\_est}}^{*}\}$ non-destructively by using Eqs. (10) and (11).

Additionally, in this study, measured strains on the weld metal after machined $\{\boldsymbol{\varepsilon}_{\mathrm{wam}}\}$ are added to Eqs. (10) and (11) to increase measurement information as shown below:

$$\{\boldsymbol{\varepsilon}_{\mathrm{est}}^{*}\boldsymbol{\varepsilon}_{\mathrm{p\_est}}^{*}\}^{\mathrm{T}} = [\boldsymbol{R}_{\mathrm{abw}}]^{+}\{\boldsymbol{\varepsilon}_{\mathrm{ebm}}\boldsymbol{\varepsilon}_{\mathrm{eam}}\boldsymbol{\varepsilon}_{\mathrm{wam}}\}^{\mathrm{T}} \tag{12}$$

$$[\boldsymbol{R}_{\mathrm{abw}}] = \begin{bmatrix} \boldsymbol{R}_{\mathrm{b}} & \boldsymbol{0} \\ \boldsymbol{R}_{\mathrm{a}} & \boldsymbol{R}_{\mathrm{a}} \\ \boldsymbol{R}_{\mathrm{a}} & \boldsymbol{R}_{\mathrm{a}} \end{bmatrix} \tag{13}$$

Note that residual strains on surface can be obtained non-destructively by the EBSD (electron backscatter diffraction patterns) method [Wilkinson (1996)] instead of X-ray diffraction.

Numerical Simulation

In this study, numerical simulation in the bead flush method based on the eigen-strain method is conducted to show the effectiveness of this method.

FEM Model

As shown in Fig. 2, a half of a butt-welded plate without geometrical restrictions at both ends was used as FEM model. The plate length, thickness and width are 120mm, 10mm and 60mm, respectively. The bead width is 8mm and its height is 0.3mm. Solid element that has 8 nodes and 3 degrees of freedom were applied. The total nodes and elements of the model are 3349 and 2544, respectively. Young's modulus and Poisson's ratio were set at 200GPa and 0.265, respectively. A commercial software, ANSYS (CYBERNET SYSTEMS CO., LTD., Japan), was used here.

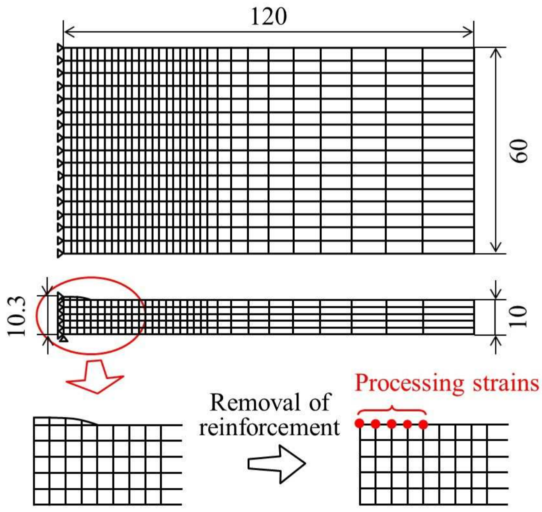


Figure 2. FEM model [Ogawa (2013)]

Exact Distribution

Exact eigen-strain distributions assumed in this simulation were quoted from the research report by Kumagai et al. [Kumagai et al. (1999)] in which eigen-strains were determined on the basis of the experimental results in the cutting method (Fig. 3). Here, the exact eigen-strains are distributed uniformly in the welding and thickness directions. Three dimensional exact residual stress distributions can be calculated from exact eigen-strains by elastic FEM analysis. For example, exact stresses at the middle in the welding line on the bottom surface (x=30mm and z=0mm) are seen in Fig. 4. Here, x, y and z directions are the welding, perpendicular the welding and thickness directions, respectively.

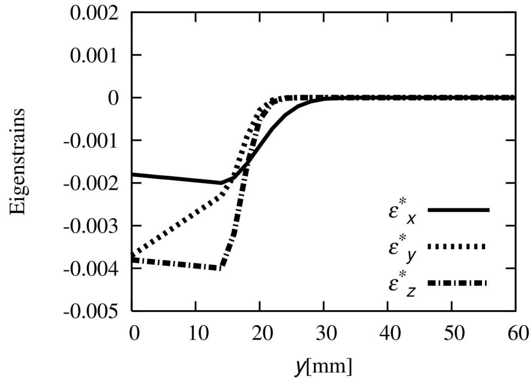


Figure 3. Exact eigen-strain distributions [Ogawa (2013)]

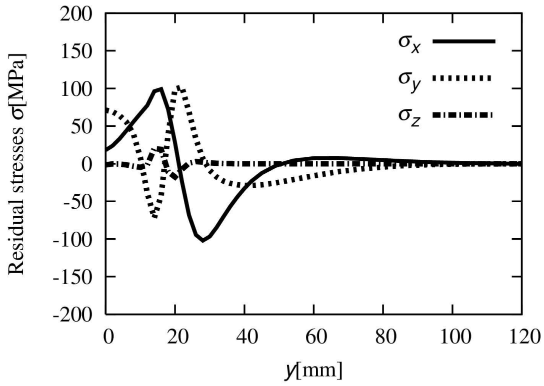


Figure 4. Exact residual stress distributions (x=30mm on the bottom surface) [Ogawa (2013)]

Procedure to Evaluate Estimation Accuracy

First, exact residual strain distributions for the whole structure are calculated by inputting exact eigen-strains in the FEM model as initial strains. Second, measured residual strains in the x and y directions at measurement points are obtained by adding measurement errors to exact residual strains. Here, measurement points on the top surface (z=10mm) on the base metal and weld metal are shown in Fig. 5 and Fig. 6, respectively. Third, estimated eigen-strains were computed by an inverse analysis. And, residual stresses on the bottom surface (z=0mm) are calculated to compare exact residual stresses. In this analysis, -500μ eigen-strains in the x direction were added evenly on the machined surface as processing strains. It is based on the assumption that micro cutter was used to remove the reinforcement of the weld [Chen et al. (1996)].

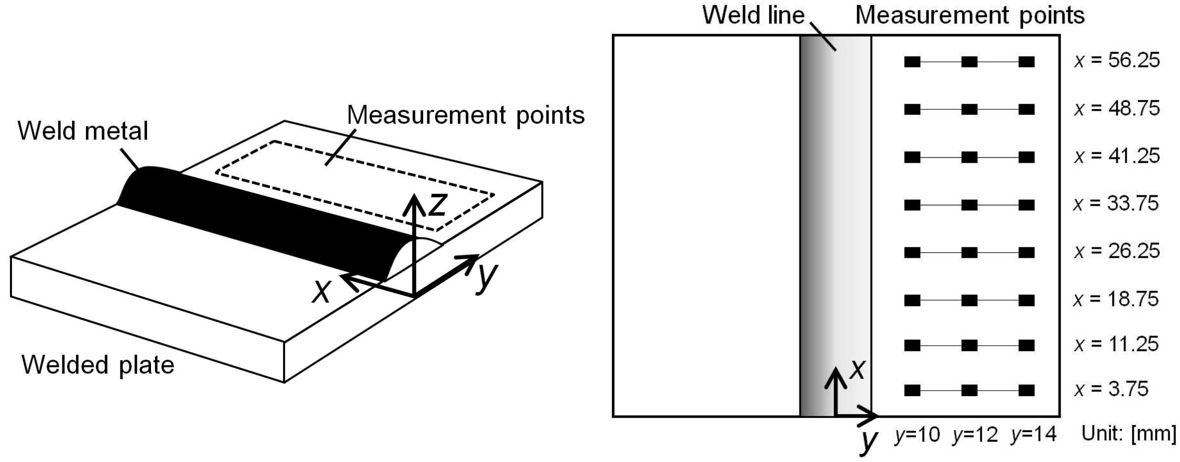


Figure 5. Measurement points on the base metal

Stabilization of Solution in Inverse Analysis

In order to reduce unknown parameters in this inverse analysis, welding eigen-strain distributions in each direction were expressed by the four logistic functions [Kumagai et al. (1999)] as:

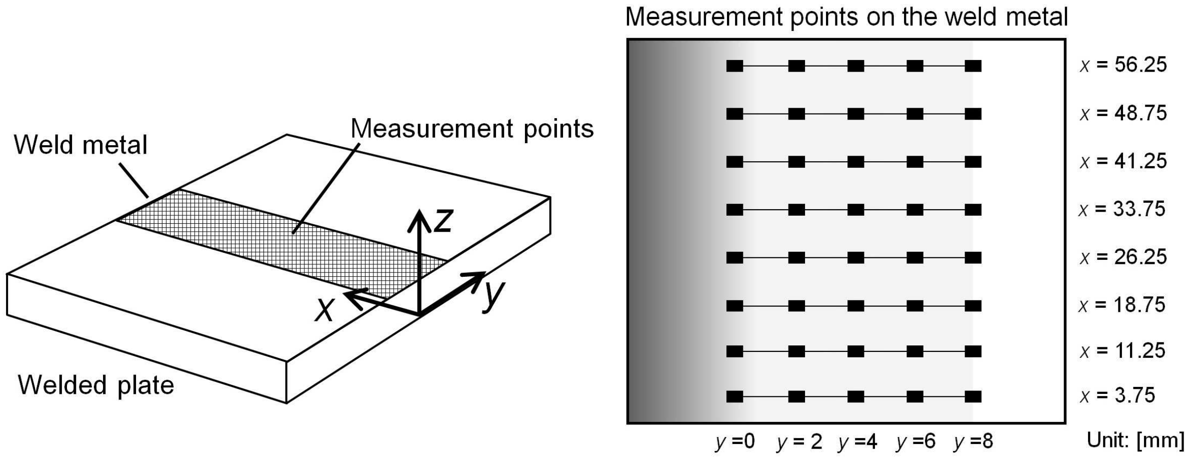


Figure 6. Measurement points on the weld metal

$$\{\boldsymbol{\varepsilon}_s^*\}(y) = \sum_{i=1}^{4} \frac{\{\boldsymbol{a}_{si}\}}{1+\exp(p+q_i y)} \tag{14}$$

$$p = -5.0\,,\ q_1 = 0.60\,,\ q_2 = 0.40\,,\ q_3 = 0.30\,,\ q_4 = 0.25 \tag{15}$$

where the subscript s denotes x, y and z directions, respectively. Constant values p and q_i were determined in consideration of that welding eigen-strains were distributed less than 40mm in the y direction [Ueda et al. (1993)]. $\{\boldsymbol{a}_{si}\}$ is a vector of unknown parameters. In addition, it was assumed that eigen-strains were constant in the welding and the thickness directions. Therefore, total number of unknown parameters of welding eigen-strains becomes twelve (4 functions × 3 directions). Furthermore, processing strains on the machined surface were considered as constant in the welding direction. The total number of unknown parameter becomes fifteen (5 points in each direction).

In order to stabilize solutions, the artificial noise method was used [Ogawa and Nakamura (2011b)]. When $[\boldsymbol{R}_{\mathrm{abw}}]$ in Eq. (12) is an $N{\times}M$ matrix with rank n, it can be decomposed as follows:

$$[\boldsymbol{R}_{\mathrm{abw}}]^{+} = [\boldsymbol{U}][\boldsymbol{B}]^{-}[\boldsymbol{V}]^{\mathrm{T}} \tag{16}$$

$$[\boldsymbol{B}]^{-} = \begin{bmatrix} \boldsymbol{B}_n^{-} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix},\ [\boldsymbol{B}_n]^{-} = \begin{bmatrix} 1/\mu_1 & & & 0 \\ & 1/\mu_2 & & \\ & & \ddots & \\ 0 & & & 1/\mu_n \end{bmatrix},\ \mu_1 \geq \mu_2 \geq \cdots \geq \mu_n \geq 0 \tag{17}$$

where $[\boldsymbol{U}]$, $[\boldsymbol{B}]$ and $[\boldsymbol{V}]^{\mathrm{T}}$ are $N{\times}N$, $N{\times}M$ and $M{\times}M$ matrixes, respectively. The values of μ_j ($1 \leqq j \leqq n$) are termed as singular values of $[\boldsymbol{B}]$. Solutions become sensitive if singular values are smaller.

In the artificial noise method, $[\boldsymbol{B}_n]^{-}$ matrix is replaced by $[\boldsymbol{B}_{n\_\gamma}]^{-}$ as shown below:

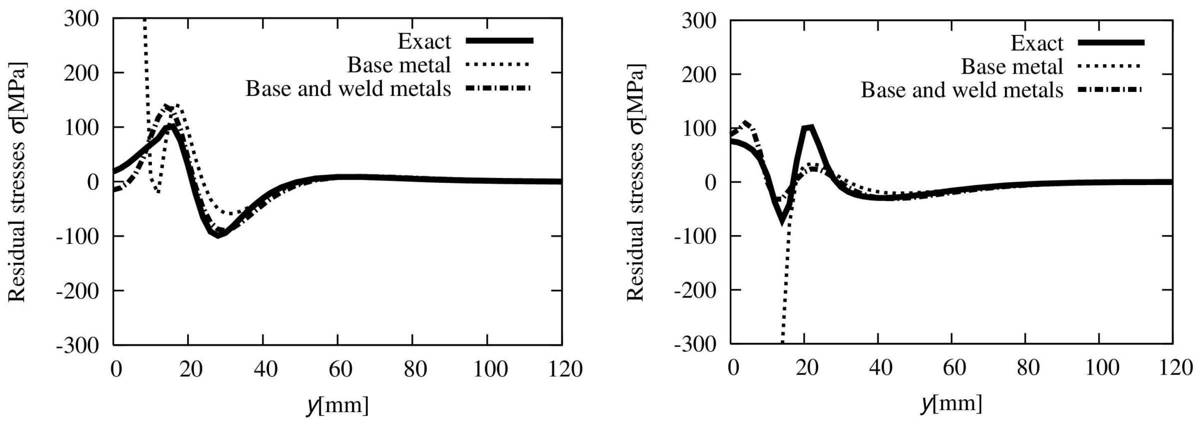
$$[\boldsymbol{B}_{n\_\gamma}]^{-} = ([\boldsymbol{B}]^2 + \gamma[\boldsymbol{I}])^{-}[\boldsymbol{B}]^{\mathrm{T}} \tag{18}$$

$$= \begin{bmatrix} \dfrac{\mu_1}{\mu_1^2+\gamma} & & & 0 \\ & \dfrac{\mu_2}{\mu_2^2+\gamma} & & \\ & & \ddots & \\ 0 & & & \dfrac{\mu_n}{\mu_n^2+\gamma} \end{bmatrix} \tag{19}$$

where [$\boldsymbol{I}$] is unit matrix. Solutions can be stabilized by increasing the real parameter γ.

Results and Discussion

Figure 7 shows estimation accuracy of welding residual stresses on the bottom surface from residual strains on the base metal (Fig. 5) and on the base and weld metals (Figs. 5 and 6). In this analysis, it was assumed that observation error follows the normal distribution whose average was 0 and standard deviation was set as 500μ because measurement accuracy of X-ray diffraction for welded joints was about ±100MPa [Kurimura and Akiniwa (2009)]. As we can see in Fig. 7, welding residual stresses cannot be estimated accurately from residual strains just on base metal. On the other hand, stress evaluation accuracy can be improved successfully when measured strains on the weld metal are used as additional source of information. Here, the L-curve method [Hansen (1992)] was used to determine the value of the artificial noise.



(a) Residual stresses in the x direction (b) Residual stresses in the y direction

Figure 7. Estimated residual stresses on the bottom surface after the removal. The dotted and chain lines are estimated results from residual strains on the base metal and on the base and weld metals, respectively.

Conclusions

In the previous study, the author developed the bead flush method that is free from the influence of processing strains in machining. However, stress evaluation accuracy is relatively poor especially in the vicinity of the weld line when measurement errors by X-ray diffraction are considered. In this study, mathematical expressions to be able to use measured strains on weld metal as additional source of information was shown. And, numerical simulation for butt welded plate was carried out to prove the effectiveness of this method. It was clarified that estimation accuracy of residual stresses especially near the weld line could be improved successfully by using this method.

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